Control of Physical Contact and Dynamic Interaction

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1 Motivation

This paper considers the influence of physical contact and mechanical interaction on the dynamics and control of manipulators. Manipulation fundamentally requires contact with the object(s) being manipulated; contact implies mechanical interaction; and mechanical interaction can have a profound influence on manipulator dynamics and control – for example, controller stability is easily jeopardized. In many present robot applications the dynamic effects of physical contact may be neglected either because contact forces are relatively small or because they need not be controlled accurately. However, the dynamics of physical contact are likely to become more prominent as newer robot designs permit relatively larger payloads and robots are applied to tasks requiring more precise control of mechanical interaction, e.g., applications involving intimate physical interaction with humans such as haptic virtual environments or personal-care robots.

This paper will review several different approaches to controlling mechanical interaction, including impedance and admittance control, and compare them to hybrid force-position control. The problems of physical contact and the strengths and weaknesses of interaction control will be considered under two broad headings: (1) implementation and (2) specification of a desired behavior for a given task.

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2 Current Approaches

2.1 Regulator Design

The majority of the vast literature on control system design has been dominated by the problem of regulation or tracking. Control authority is exerted to ensure that some variable of interest is kept close (in some appropriately defined sense) to a desired value despite uncertainties and hardware limitations. From that perspective, contact tasks are often assumed to require an appropriately coordinated hybrid of position control and force control. In many applications regulator design has been remarkably effective and it has been refined to a high degree of sophistication but it is not necessarily the best approach to all problems. Applied to physically interacting systems, it can be misleading.

From a regulator design perspective, interaction between a control system and its environment causes a "disturbance" that should be counteracted. Disturbances are usually assumed to be small in some sense (e.g., have less influence on system behavior than the available control effort); are typically assumed to have a frequency content distinct from that of the variable to be tracked; and are typically assumed to be independent of the state of the system. The consequences of physical contact commonly satisfy none of these conditions. Consider two persons shaking hands: each can exert comparable forces, so the "disturbance" due to contact is unlikely to be small; each has a comparable bandwidth for voluntary movement, so the frequency content of the "disturbance" is likely to be the same as that of the desired behavior, not distinct from it; and far from being independent, the "disturbance" experienced by each hand is largely due to its own actions. It may seem that with a sufficiently detailed model of the environment these problems could be avoided but the behavior of the environment may be arbitrarily complex and very poorly known. For example, in its sensory role, the human hand is frequently used to explore objects and identify their dynamic behavior. By definition these objects have not been modeled a priori, yet the hand remains well-controlled during physical contact and exhibits no pathological behavior. As reviewed below, design constraints have been identified that permit a robot controller to perform satisfactorily during physical contact with objects of arbitrarily complex and almost completely unknown dynamic behavior.

2.2 Interactive Behavior

Perhaps the most important limitation of the regulator/tracker design approach is its focus on the "forward-path" response to a command or reference input. In situations involving physical contact, response to the environment may be at least as important as response to a reference input, perhaps more so. An example may be found in "haptic virtual environments", computer-driven, human-interactive displays to synthesize the illusion of a (software- generated) environment. Visual and acoustic displays dominate current virtual environment technologies, but there is a clear need for *haptic* displays to synthesize the touch and feel of contact with objects. A robot with a suitable means of coupling to a human can serve as a programmable haptic display and some sophisticated designs have recently emerged.

The design of the controller for a haptic display device presents an interesting challenge. If it is approached as a regulator design problem, what should be the reference input? While the hand is not in contact with a virtual object, zero force may be required. A force regulator might therefore seem appropriate, but how can the boundary of a virtual object be described in terms of a reference force? Within the region occupied by a virtual object, hand motion should be opposed or even prevented. This might seem to call for a position regulator, but how can the arbitrary motions permissible when not in contact be described in terms of a reference position? It is simpler to regard the controller as implementing a (specified) relation between force and position (or motion), a mechanical impedance, than as regulating force or position. When not in contact, impedance should be low, probably zero. Within the region occupied by a virtual object, impedance should increase to reflect how the object resists deformation.

2.3 An Interaction-Port Perspective

Given that a manipulator may contact a wide variety of objects, some with largely unknown dynamics, it is useful to direct the controller design toward those properties of a manipulator that do not change when it contacts objects. One such property is how it feels from the "outside"; the behavior of the manipulator exhibited at the interaction port by which it couples to its environment.

We define an *interaction port* by a set of variables that describe the exchange of energy between a system and its environment. Typically these are the forces and positions at points of contact but the idea is readily extended to other variables and to non- contact forms of energetic interaction. The definition of the variables requires care: each force or moment (more generally, *effort*) must be associated with a corresponding position or angle (more generally, *displacement*) and its rate of change (velocity, angular rate or, more generally, *flow*) such that each effort-flow pair (known as *power conjugate variables*) properly defines a power flow by which the system energy may change.[1]

The *interaction port behavior* determines a map relating the port variables. Borrowing terms that originated in electrical network theory, *impedance* relates motion to force and *admittance* relates force to motion. Impedance and admittance are commonly used as frequency domain descriptions of linear systems, but they are readily generalized to nonlinear systems. More precisely, impedance is a causal dynamic operator that maps an input motion time function $\dot{x}(t)$ to an output force time function $F(t)$; admittance is the dual operator relating $F(t)$ to $\dot{x}(t)$. In the linear case one is the inverse of the other; in the nonlinear case the inverse operators may not be definable. For example, assuming a state-determined repre-

[1] The generalized efforts and generalized velocities used in mechanics are an example of an effort-flow pair for which a power flow is defined. Less commonly used wrenches and screws provide another example.

sentation of system dynamics, impedance is described by rate equations $\dot{z} = Z_r(z, \dot{x}, t)$ and output equations $F = Z_o(z, \dot{x}, t)$, where z is a finite-dimensional state vector and $Z_r()$ and $Z_o()$ are algebraic (memoryless) functions. Impedance (or admittance) should not be confused with its more common parameterizations. It is more general than a combination of mass, friction and elasticity; an impedance may be described by arbitrarily complex, nonlinear dynamics and there are many alternative parameterizations of the map from motion to force.

Impedance control is a generic term like motion control or force control. Impedance determines interactive behavior by definition; the impedance control approach is to characterize, modulate and (to the extent possible) determine the impedance at the port(s) of interaction with the environment. Impedance control should not be confused with its particular implementations. It is not confined to position and velocity feedback control of rigid-member inertial mechanisms actuated by current-controlled torque motors; in its most general form it is not simply emulation of mass-spring-damper systems. The main distinction of impedance control is that it attempts to govern the interactive behavior of a manipulator that is unaffected by contact with objects. It is often convenient to think of mechanical impedance as a dynamic generalization of stiffness. Stiffness at a point is a system property independent of any objects it may contact. In contrast, the force at that point is strongly affected by contact with objects. Force control (and likewise motion control) is therefore fundamentally sensitive to object contact; impedance control need not be.

3 Contact Instability

Physical contact and mechanical interaction can have a profound affect on control system stability. Attempts to control the force exerted by a robot have been thwarted by the phenomenon of contact instability. A machine that can stably execute unrestrained motions may become unstable on contact with an object. Whitney [29], [30] first reported the problem with robot force feedback control. He observed that the stiffness of the contacted surface plays a role analogous to a feedback gain, hence stable contact imposed a limit on the stiffness of the surface or the stiffness of the robot at the point of contact. Despite the work of numerous researchers, the problem proved to be remarkably refractory and was regarded as a fundamental challenge of robotics by Paul [22].

Several workable solutions emerged from subsequent work. One important factor is that in a typical configuration, significant dynamics (e.g., due to flexibility of the robot joints or members or to sensor dynamics) are interposed between the force feedback sensors and the robot actuators; the sensors and actuators are *non-collocated*. A force feedback control loop (which is not closed until contact) excites the interposed dynamics and its maximum stable gain is severely limited by their presence. One way to avoid this problem is to minimize robot structural dynamics and significantly improved designs (e.g., using composite materials) have been developed. Another class of solutions is to minimize the non-collocation problem using small, fast end- effectors near the point(s) of contact [27], [24], [25].

However, it is important to recognize that contact instability is not solely due to non-collocation of sensors and actuators. Indeed, contrary to common misperception, contact instability is not confined to force feedback controllers contacting stiff surfaces. It is a more general problem that can re-surface despite improvements in robot design as new and more demanding applications are attempted. For example, most of the devices presently being used as haptic displays are prone to contact instability for certain classes of emulated (virtual) objects or as the human operator changes grip strength.

3.1 Interactive Stability and Passivity

An interaction port perspective facilitates a general approach to the contact instability problem and has been used to identify conditions for preserving stability on contact that are both sufficient and necessary (for an appropriately defined class of objects). A large class of objects a manipulator may encounter are passive. They can store and return energy, but cannot generate energy. This is important because passivity is widely used in control theory for stability analysis. The stability of adaptive controllers can be established using a passivity argument in which an energy storage function is used as a candidate Lyapunov function [19], [21]. Using an input-output analysis, Cho and Narendra [3] showed that a negative feedback interconnection of strictly passive operators is sufficient to guarantee stability.[2]

When two objects are coupled at an interaction port they may exchange energy but the connection itself cannot generate energy. This means that the mathematical operators describing their interaction port behavior are restricted in the ways they may be combined. If the interaction port is described using force and velocity, the interaction port behavior of each object is an impedance or admittance. If two objects are coupled their interaction is equivalent to a unity gain negative feedback connection of their respective impedance and admittance. For example if the velocity at a point of contact is common to both objects, then by Newtons third law, the forces on each object at the contact point are equal and opposite. As a result, a *sufficient* condition to preserve stability on contact with a passive object is that the manipulators impedance (or admittance) should appear to be passive.

Using a linear analysis, the necessity of this condition may be deduced by considering the class of all passive objects. A sketch of the argument is presented here; details are in [8], [9]. Linearizing the dynamic equations about the nominal contact conditions, at steady state in response to sinusoidal excitation both the force and velocity at the contact point will be sinusoids. Considering the entire class of passive objects, the relative magnitudes of the two sinusoids are unrestricted but the relative phase angle between the two may not exceed $\pm 90°$ because passivity requires that the average power transmitted to the object must be non-negative.

Because physical contact of two systems is equivalent to a feedback connection of their impedance and admittance operators, the Nyquist criterion may be used for stability analysis. The necessary and sufficient condition for stable interaction is that the Nyquist plot of the product of the manipulator impedance and object admittance (or vice versa) must not encircle the -1 point on the complex plane. The Nyquist plot of the product may be obtained by rotating the Nyquist plot of the manipulator impedance through the angle determined by the Nyquist plot of the object admittance, which can be no more than $\pm 90°$, and multiplying it by the magnitude of the Nyquist plot of the object admittance. Because the magnitude of the object admittance can take on any value the necessary and sufficient condition for stability of the coupled system is that the Nyquist plot of the manipulator impedance must lie exclusively in the right half of the complex plane; but that means that the manipulator impedance must be a positive-real function, i.e., its interaction port behavior must be indistinguishable from that of a passive object.

3.2 Conservatism and the Practicality of Passivity

A common objection to passivity-based analysis of control system stability is that it yields sufficient conditions that may be excessively conservative. Therefore the claim that a passive interaction port behavior is necessary as well as sufficient should be examined carefully. First of all, the necessary condition was established assuming a linearized representation of interaction port behavior and consequently it is only as true as the fidelity of the linearized model, i.e., it is a local result valid in the vicinity of the nominal operating conditions. For nonlinear systems, passive interaction port behavior is sufficient to ensure contact stability (e.g., [8], [12], [13], [14]) but a global necessary condition has yet to be established.

Second, the necessity of passive interaction port behavior for linearized systems arises from the assumed class of objects: all possible passive objects. That is an extremely broad class that includes infinitely stiff and infinitely massive objects. In any realistic application the class of objects is likely to be more restricted (e.g., the stiffness or mass of objects contacted may be bounded) and it may be possible to use knowledge about that restricted class of objects to relax the coupled stability condition. Despite these caveats the contact stability condition provides useful insight for practical applications. In one early experiment using a simple two- link robot as a test platform, Colgate [5], [9] designed a control system to make its end-point follow a commanded trajectory in a small region in the workspace center. The controller used only position and velocity feedback and was designed using the LQG/LTR technique, a widely-recognized modern approach to multi-variable controller design. That technique does not constrain the interaction port behavior to be passive and, in fact, both analytical prediction and direct measurement on the hardware showed that the impedance due to the LQG/LTR controller violated the condition for contact stability. The passivity analysis predicted that instability would occur on contact

[2] A rigorous definition and detailed discussion of the several definitions passivity may be found in [31], [7].

with certain classes of objects including both sufficiently stiff springs and sufficiently large masses. Experimental observation confirmed these predictions; although position was well-controlled when unconstrained, the robot was unstable on contact with a stiffness of 420 (± 20) N/m and mass of 1.4 (± 0.1) kg.

Note that in addition to confirming the value of the passivity analysis, this experiment clearly demonstrated that contact instability is not merely a quirk of force-feedback controlled robots contacting rigid surfaces. It is a fundamental problem of contact; it may occur in the absence of force feedback and while contacting unconstrained objects. Nor is this a peculiarity of the LQG/LTR technique: PID controllers are commonly used to control robot joint position but they violate the passivity condition by producing excessive phase lag between (disturbance) force and motion at low frequencies. The analysis predicts that these controllers will exhibit instability when coupled to sufficiently large unconstrained masses and this was also confirmed experimentally [9]. This kind of instability may become an important consideration if robots are to be used to manipulate large unconstrained masses (e.g., in space or underwater applications).

3.3 Passivity of Discrete-Time Controllers

An important limitation of the passivity-based condition for contact stability is that it constrains the phase of the interaction port behavior at all frequencies. If a discrete-time implementation is used then the delay due to data sampling introduces a phase lag that grows without bound at high frequency and this violates the passivity condition. Given the overwhelming advantages of digital control this would seem to limit the usefulness of a contact stability condition that requires passivity. This motivated Chapel and Su [2] to investigate "nearly passive" robots.

In fact, recent work by Colgate [6] has dramatically reduced the conservativeness of the coupled stability criteria and Colgate and Schenkel [7] have identified conditions that guarantee the passivity of machines under discrete control. An elegant solution is to include a physical energy dissipator (e.g., a linear damper) at the interaction port that serves to remove energy added by the active behavior of the discrete-time controller. Loosely speaking, this physical energy dissipator may be considered analogous to the use of an anti-aliasing filter in an analog-to-digital converter. It ensures that the interaction between the continuous domain of physical variables and the discrete domain of digital computation does not lead to pathological behavior.

3.4 A Small-Gain Approach to Interactive Stability

The problem of contact instability can be approached from a different perspective. Kazerooni [18] (see also [16], [17], [28]) presents an analysis (with corroborating experiments) that uses the small-gain theorem to derive a sufficient condition for stable compliant motion using force feedback and a controller design procedure is developed from that sufficient condition. The permissible force feedback gain is upper-bounded by a quantity that decreases as either the robot or the contact surface become stiffer, a result consistent with widespread experimental observation.

This analysis has certain features that limit its generality. First, it is assumed that a force feedback loop is closed around a position-controlled robot. The analysis does not apply to alternative control architectures, e.g., motion feedback around a high- gain force/torque controller. Second, it is assumed that, in the absence of force feedback, the position- controlled robot is stable when in dynamic contact with its environment. Position control is certainly common in robot applications, but because stability is assumed at the outset, the analysis cannot identify any constraints on the position controller to ensure stable contact.

Third, the approach describes both the displacement response of the robot to contact forces and the environmental forces evoked by robot displacements using operators that have magnitude-bounded gains. This means that certain classes of robot position control cannot be considered, including the commonplace PID position control which does not have a bounded magnitude at DC. Also, infinitely stiff surfaces cannot be considered, even though that may be a practical description of the kinematic constraint due to contact.

In contrast, the passivity-based analysis restricts only the *phase* of the operators used to relate contact forces and motions; their magnitudes may be arbitrary. Given that a manipulator may have to contact completely unfamiliar objects a description that applies to infinitely stiff or infinitely compliant objects seems preferable. Given that some applications may require the application of large forces a stability condition that does not restrict the magnitude of the robot reaction seems preferable.

Finally, note that the result derived from the application of the small-gain theorem to a force controlled robot yields a gain constraint that changes with the stiffness of the contacted object. This is because contact force is not a property of the manipulator nor of the object. On the other hand, the passivity analysis yields a constraint on a manipulators interaction port behavior, and that is a property of the manipulator independent of any objects that are contacted.

4 Computational Complexity

One of the main impediments to using interaction controllers (e.g., impedance, admittance, compliance or stiffness controllers [4], [15], [20], [23]) is that force or motion at the interface are not explicitly controlled but implicitly determined by the interaction port behavior. In practice it has proven difficult to select appropriate impedance parameters to execute useful tasks. One important reason is that the relation between interaction port behavior (e.g. at a robots end-effector) and the corresponding actuator behavior (e.g., in robot joint coordinates) is extremely complex, a highly nonlinear function of the geometric and kinematic properties of the manipulator. This problem is particularly acute for spatial manipulators which involve the complex kinematics of spatial rotations.

The stiffness of a manipulator at its end-effector defines a static relation between end-effector configuration and wrenches (torques and forces). Let ϕ be a chart mapping end-effector configurations to generalized coordinates with $\phi(q) = x_r$. Translation is commonly represented using Cartesian coordinates; orientation is commonly represented using angles such as roll, pitch and yaw angles. Associated with generalized coordinates, x_r, are generalized velocities, $\dot{x}_r$, and generalized forces, F. Stiffness is then a static relation between x_r and F. A commonly used relation is

$$F = K_\phi (x_r - x_v), \tag{1}$$

where K_ϕ is a stiffness matrix, x_r is an array of coordinates of the robot end-effector configuration and x_v is an array of coordinates of the so-called virtual equilibrium configuration. The configuration-wrench behavior of this apparently simple stiffness is remarkably complex. It is highly dependent on the manipulator and equilibrium configurations and as a result it is inordinately difficult to select appropriate stiffness parameters.

This problem is further compounded by the transformation of interaction port behavior into a coordinate frame relevant to a task, which is similarly complex and nonlinear, especially for spatial tasks. That means that the already complex mapping of task-relevant behavior into manipulator behavior may change significantly in different phases of a task and that makes it difficult to break a complex application down into manageable parts.

Recent work [10], [11] has shown how the interaction port behavior may be decomposed into "spatial" and "nonspatial" parameters. "Spatial" parameters, e.g., principal directions of translational stiffness, have well-defined, intuitive, spatial transformation properties and can be selected depending on the configuration of objects with which the robot must interact. "Nonspatial" parameters, e.g., principal translational stiffnesses, can be selected independently of object configurations. This decomposition dramatically simplifies the specification of impedance parameters.

The approach is summarized in the following sections which analyze the spatial transformation of manipulator compliance, part of the impedance selection problem, and describe a *spatial compliance controller* for a serial, rigid manipulator. The results generalize readily to other impedance parameters [11].

5 A Spatial Compliance Controller

The configuration of a rigid body can be represented by a frame attached to the body. A *frame* is a quadruplet (p, e_1, e_2, e_3), where p is a displacement vector representing position and vectors e_1, e_2 and e_3 are an orthonormal triplet representing orientation. The vectors are described with respect to a fixed, orthonormal reference frame. Each frame can be identified with a homogeneous matrix:

$$H = \begin{bmatrix} R & p \\ 0^t & 1 \end{bmatrix}, \tag{2}$$

where $R = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}$ is an orthonormal matrix.

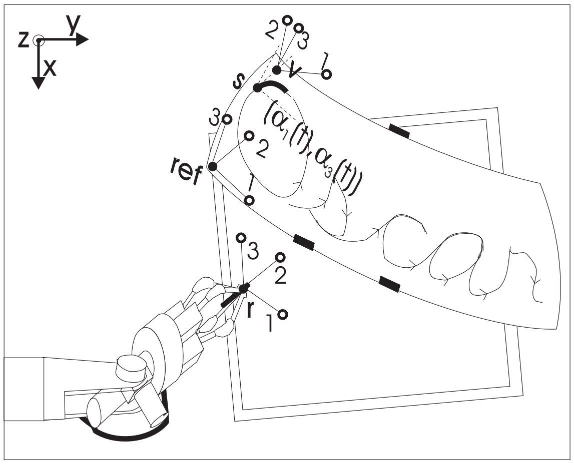

Figure 1: Interaction with a spherical surface. Compliance acts to align the robot and virtual equilibrium frames.

Shown in Fig. 1 are three frames. Frame $(p, e_1, e_2, e_3)_r$ is attached to the end-effector of the robot. Frame $(p, e_1, e_2, e_3)_s$ is attached to a body of interest, in this case a spherical surface. Frame $(p, e_1, e_2, e_3)_v$ represents the virtual equilibrium configuration of compliance.

"Spatial" compliance acts to align the end-effector frame and the virtual equilibrium frame, and is simply described in terms of its potential energy:

$$\begin{aligned} U &= \tfrac{1}{2}\Delta p^t S \Lambda_t S^t \Delta p - \operatorname{tr}\left(\Lambda_o R_v^t R_r\right) \\ &= \tfrac{1}{2}\Delta p^t K \delta p - \textstyle\sum_{i=1}^{3} \kappa_i e_{ir}^t e_{iv}, \end{aligned} \qquad (3)$$

where $\Delta p = p_r - p_v$ and where $\operatorname{tr}(A)$ is the trace of matrix A. This energy function defines a family of potential energy functions parameterized by S, H_v, Λ_t and Λ_o. Parameter $S = [\ v_1 \quad v_2 \quad v_3\ ]$ is an orthonormal matrix, parameter Λ_t is a diagonal matrix with elements k_1, k_2 and k_3, and parameter Λ_o is a diagonal matrix with elements κ_1, κ_2 and κ_3.

The first energy term is that of a linear, translational spring of stiffness $K = S\Lambda_t S^t$, which acts to coincide points p_r and p_v. The principal directions of stiffness are v_1, v_2 and v_3. The corresponding principal stiffnesses are k_1, k_2 and k_3. The second energy term is that of a rotational spring that acts to align R_r and R_v.

The compliance associated with each $-\kappa_i e_{ir}^t e_{iv}$ term acts to align corresponding vectors e_{ir} and e_{iv}. Energy $-\kappa_i e_{ir}^t e_{iv}$ is minimized when vectors e_{ir} and e_{iv} are aligned ($e_{ir}^t e_{iv} = 1$), and maximized when they are anti-aligned ($e_{ir}^t e_{iv} = -1$). Parameters κ_1, κ_2 and κ_3 are orientational compliance parameters referred to here as orientational stiffnesses. In the sequel it is shown that these parameters determine the effective rotational stiffnesses for small displacements.

5.1 Control Law Based on Kinestatic Robot Model

It is straightforward to derive a corresponding control law for serial manipulators. Assume that the generalized coordinates representing robot joint configuration are $\theta = \{\theta_i\}$ and that the corresponding generalized forces are $\tau = \{\tau_i\}$. If static friction, gravity, link compliance, etc., can be neglected in the manipulator kinestatic model, taking the partial derivative of energy (Eqn. 3) with respect to the generalized coordinates yields a suitable control law:

$$\tau = \left(\frac{\partial p_r}{\partial \theta}\right)^t K(p_r - p_v) - \sum_{i=1}^{3} \kappa_i \left(\frac{\partial e_{ir}}{\partial \theta}\right)^t e_{iv}, \quad (4)$$

in which each $\partial a / \partial \theta$ for $a \in \{p_r, e_{1r}, e_{2r}, e_{3r}\}$ is a 3×6 Jacobian-like matrix. This assumes that the generalized efforts are positive when work is done on the robot. In practice it is necessary to add other control terms to compensate friction and gravity, and to dissipate energy. This control law was simulated and implemented by Bonnes and Colard [1] and Tigchelaar [26]. The stability properties of the controlled robot are investigated in [11].

5.2 Dependency of Wrenches on End-Effector Configuration

The end-effector wrenches for a particular end-effector configuration are determined by the differential of the energy function at that configuration. Let δH_r be an arbitrary, infinitesimal displacement from a particular configuration, H_r, so that

$$H_r + \delta H_r = \begin{bmatrix} R_r + R_r \delta\tilde{\theta} & p_r + R_r \delta p \\ 0^t & 1 \end{bmatrix}. \quad (5)$$

In this expression, $\delta\theta$ is an infinitesimal rotation expressed in the end-effector frame and δp is an infinitesimal translation expressed in the end-effector frame. Associated with $\delta\theta$ is an antisymmetric matrix:

$$\delta\tilde{\theta} = \begin{bmatrix} 0 & -\delta\theta_3 & \delta\theta_2 \\ \delta\theta_3 & 0 & -\delta\theta_1 \\ -\delta\theta_2 & \delta\theta_1 & 0 \end{bmatrix} \qquad (6)$$

given $\delta\theta = [\ \delta\theta_1 \quad \delta\theta_2 \quad \delta\theta_3\ ]^t$. This notation shall be used to associate antisymmetric matrices and vectors. The infinitesimal work given δH_r can be computed from Eqn. 3, yielding

$$\delta W = \Delta p^t S \Lambda_t S^t R_r \delta p + \frac{1}{2}\mathrm{tr}\left(2\mathrm{as}\left(\Lambda_o R_v^t R_r\right)^t \delta\tilde{\theta}\right), \tag{7}$$

where as(A) is the antisymmetric part of matrix A. The resultant wrench expressed in the end-effector frame is

$$\tilde{m} = 2\mathrm{as}\left(\Lambda_o R_v^t R_r\right) \text{ and } f = R_r^t S \Lambda_t S^t (p_r - p_v), \tag{8}$$

because the infinitesimal work given an arbitrary, infinitesimal displacement δH_r is

$$\delta W = f^t \delta p + \frac{1}{2}\mathrm{tr}\left(\tilde{m}^t \delta\tilde{\theta}\right). \tag{9}$$

5.3 Wrenches Resulting from Small Displacements

For small displacements of the end-effector from the virtual equilibrium configuration, the wrench-configuration relation of Eqn. 8 can be approximated by a linear relation

$$\begin{bmatrix} m \\ f \end{bmatrix} \approx K \begin{bmatrix} \delta\theta \\ \delta p \end{bmatrix}, \text{ assuming} \tag{10}$$

$$H_r \approx \begin{bmatrix} R_v + R_v\delta\tilde{\theta} & p_v + R_v\delta p \\ 0^t & 1 \end{bmatrix}. \tag{11}$$

The linear approximation is computed by substituting Eqn. 11 into Eqn. 8 and discarding high-order terms, resulting in

$$f \approx R_v^t S \Lambda_t S^t R_v \delta p, \text{ and } m \approx \left(\mathrm{tr}(\Lambda_o) I - \Lambda_o\right) \delta\theta. \tag{12}$$

The corresponding stiffness matrix is

$$K = \begin{bmatrix} \mathrm{tr}(\Lambda_o) I - \Lambda_o & 0 \\ 0 & R_v^t S \Lambda_t S^t R_v \end{bmatrix}. \tag{13}$$

Expansion of Eqn. 13 shows that for H_r near H_v, $\kappa_2 + \kappa_3$ is the effective rotational stiffness about axis e_{1v}, $\kappa_3 + \kappa_1$ is the effective rotational stiffness about axis e_{2v}, and $\kappa_1 + \kappa_2$ is the effective rotational stiffness about axis e_{3v}.

5.4 Spatial Transformation of Compliance

Compliance is parameterized by two kinds of parameters. *Spatial parameters* are parameters for which the action of a rigid body transformation is defined. In this case the spatial parameters are the principal directions of translational stiffness, S, and the virtual equilibrium configuration, H_v. *Nonspatial parameters* have no special properties. In this case the nonspatial parameters are the translational and orientational stiffnesses, Λ_t and Λ_o. The compliance family has a useful transformation property. Let

$$H_\sigma = \begin{bmatrix} R_\sigma & t_\sigma \\ 0^t & 1 \end{bmatrix} \tag{14}$$

represent an arbitrary rigid body transformation. Substitution of $H_\sigma H_r$, $H_\sigma H_v$ and $R_\sigma S$ for H_r, H_v, and S, respectively, in Eqn. 8 yields

$$\tilde{m} = 2\mathrm{as}(\Lambda_o R_v^t R_\sigma^t R_\sigma R_r) = 2\mathrm{as}(\Lambda_o R_v^t R_r) \text{ and} \tag{15}$$

and

$$\begin{aligned} f &= R_r^t R_\sigma^t R_\sigma S \Lambda_t S^t R_\sigma^t (R_\sigma p_r + t_\sigma - R_\sigma p_v - t_\sigma) \\ &= R_r^t S \Lambda_t S^t (p_r - p_v). \end{aligned} \tag{16}$$

This shows that if the robot configuration and all spatial parameters are subject to a rigid body transformation, then the compliant wrenches are unchanged. This transformation property greatly simplifies compliance selection. Its value is demonstrated in the following by considering the task of writing on a spherical surface.

6 Example: Writing on a Spherical Surface

Figure 1 shows a robot end-effector and a spherical surface with a known radius of curvature, r. Shown are (1) the robot end-effector frame, $(p, e_1, e_2, e_3)_r$, labelled **r** in the figure, (2) a frame normal to the surface at a point of interest, $(p, e_1, e_2, e_3)_s$, labelled **s** in the figure, (3) a fixed, reference frame on the surface, $(p, e_1, e_2, e_3)_{ref}$, labelled **ref** in the figure, and (4) the virtual equilibrium configuration frame, $(p, e_1, e_2, e_3)_v$, labelled **v** in the figure. The robot is shown holding a writing tool. The tip of the tool is assumed to coincide with p_r; the axis of the tool is assumed to coincide with e_{2r}. The reference frame on the surface is assumed either to be known priori or to be perceptible from sensory information. Point p_{ref} lies in the surface; e_{2ref} is normal to the surface.

The task is to make a stroke on the surface. This task can be decomposed into a number of subtasks: (1) moving to a point above the surface near the start of the stroke, (2) contacting the surface at the start of the stroke, (3) making the actual stroke, and (4) moving away from the surface if necessary. Consider only the subtask of making the actual stroke, assuming contact has already been made at the start of the stroke.

The stroke is described by real-valued displacement functions $\alpha_1(t)$ and $\alpha_3(t)$ having units of distance. The spherical surface is identified with a flat surface by means of a mercatorial projection, the familiar projection used in cartography. Vectors e_{1ref} and e_{3ref} can be used to generate local directions of latitude and longitude. The displacement functions describe the stroke using coordinates of latitude and longitude.

The frame normal to the surface at the desired point of contact, $(p, e_1, e_2, e_3)_s$, can be thought of as the result of first transporting frame $(p, e_1, e_2, e_3)_{ref}$ along a latitudinal segment of arclength $\alpha_1(t)$, and then along a longitudinal segment of arclength $\alpha_3(t)$. Let $c_i(t) = \cos(\alpha_i(t)/r)$ and $s_i(t) = \sin(\alpha_i(t)/r)$. The orientation of frame $(p, e_1, e_2, e_3)_s$ is

$$R_s = R_{ref} \begin{bmatrix} c_1(t) & -s_1(t)c_3(t) & -s_1(t)s_3(t) \\ s_1(t) & c_1(t)c_3(t) & c_1(t)s_3(t) \\ 0 & -s_3(t) & c_3(t) \end{bmatrix}. \tag{17}$$

The position of this frame is

$$p_s = p_{ref} + re_{2ref} - re_{2s}, \tag{18}$$

where e_{s2} is determined by Eqn. 17. Point $p_{ref} + re_{2ref}$ is the center of the sphere.

Intuitively, the principal directions of stiffness can be chosen to be aligned with the frame normal to the surface at the desired point of contact:

$$S = R_s(t). \tag{19}$$

The translational stiffnesses tangential to the surface, k_1 and k_3, are chosen to be equal and high. The translational stiffness normal to the surface, k_2, is chosen to be low. Assume that the writing tool is to be held at a constant angle of tilt with respect to the local surface, which corresponds to a common human writing strategy. The tool is to be tilted at an angle β around the e_{3s} axis. The corresponding virtual equilibrium orientation is

$$R_v = R_s \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{20}$$

Orientational stiffnesses κ_1 and κ_3 are not important in this task as near-point contact is assumed. They are assumed to be equal and low. Orientational stiffness κ_2 is assumed to have a higher value. A simple strategy for writing on a surface is to displace the virtual equilibrium position from the desired position by a distance d normal and into the surface. Using this strategy the virtual equilibrium position is

$$p_v(t) = p_{ref} + re_{2ref} - (r - d)e_{2s}(t), \tag{21}$$

where d is the desired distance of displacement. This is a reasonable strategy if both the friction of the instrument-surface interface and the inertia of the robot can be neglected. The actual equilibrium position is then $p_s(t)$.

Equations 17-21 are sufficient to describe the task independently of the surface configuration, $(p, e_1, e_2, e_3)_{ref}$. It is not claimed that this is the best nor even a good strategy for writing; the point is to show that the spatial compliance family is parameterized in such a way that it can be used to describe complex spatial interaction with bodies in the environment in a simple way. This eliminates one of the computational barriers to applying interaction controllers to interesting tasks. An assembly example is given in [11].

7 Discussion

Consider again "conventional" stiffness or compliance control. One could call the eigenvectors of K_ϕ and the coordinates of the virtual configuration, x_v, "directional parameters". One could call the eigenvalues of K_ϕ "nondirectional parameters", but as will be shown this would not be useful.

Compliance selection is often simplified by introducing "task coordinates". Let ψ be a second chart mapping end-effector configurations to generalized coordinates with $\psi(q) = y_r$. These coordinates might be three Cartesian coordinates and three angles with respect to a frame attached to a body of interest, such as the $(p, e_1, e_2, e_3)_{ref}$ frame in the example. We then have $x_r = \phi \circ \psi^{-1}(y_r)$ and $\delta y_r = J \delta x_r$, where J is a Jacobian matrix. Using chart ψ compliance can be expressed as

$$G = K_\psi(y_r - y_v) \tag{22}$$

where G is the generalized force corresponding to the generalized velocity $\dot{y}_r$. Selection of the virtual equilibrium coordinate and stiffness matrix with respect to chart ψ, i.e., selecting y_v and K_ψ, is assumed to be straightforward. The virtual equilibrium coordinate and stiffness with respect to chart ϕ are then

$$x_v = \phi \circ \psi^{-1}(y_v) \text{ and } K_\phi = J^t K_\psi J. \qquad (23)$$

If Eqn. 23 is satisfied then the compliance relations of Eqn.'s 1 and 22 generate the same configuration-wrench behavior for small displacements of the end-effector from the virtual equilibrium configuration. Matrices K_ψ and K_ϕ do not necessarily have the same eigenvalues; the eigenvalues of K_ϕ will in general depend on the configuration of the objects with which the robot interacts. In the example we were able to choose nonspatial parameters, k_1, k_2, k_3, κ_1, κ_2, κ_3, independently of the configuration of the surface, $(p, e_1, e_2, e_3)_{ref}$. The preceding discussion shows that one could not choose the eigenvalues of K_ϕ independently of the coordinates of the surface, x_s. The spatial parameters, v_1, v_2, v_3 and $(p, e_1, e_2, e_3)_v$, were related geometrically to the configuration of the surface, $(p, e_1, e_2, e_3)_{ref}$. One could not relate x_v or the eigenvectors of K_ϕ so simply to the coordinates of the surface, x_{ref}. The compliance of Eqn. 1 is not described using the relevant geometrical structure of the configuration manifold of a rigid body.

8 Future Directions

Recent developments have shown how to design and implement practical robot controllers that are robust to physical contact and dynamic interaction with a broad class of passive objects with almost arbitrarily complex dynamic behavior. A new parameterization of interaction port behavior that explicitly represents its geometrical structure simplifies the deployment of these controllers in complex, realistic tasks. Of course, numerous unsolved problems remain. One is the case of dynamic interaction with active objects, in which the actions of the manipulator may be equivalent to coupling a power source to the interaction port. An example is the use (or misuse) of certain classes of power tools. How to control physical contact and dynamic interaction in this situation is an open question. Interestingly, humans seem remarkably adept at this class of manipulation.

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