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Full paper

A globally stable state feedback controller for flexible joint robots

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Abstract—The paper addresses the problem of controlling the joints of a flexible joint robot with a state feedback controller and proposes a gradual way of extending such a controller towards the complete decoupling of the robot dynamics. The global asymptotic stability for the state feedback controller with gravity compensation is proven, followed by some theoretical remarks on its passivity propertys. By proper parameterization, the proposed controller structure can implement a position, a stiffness or a torque controller. Experimental results on the DLR lightweight robots validate the method.

Keywords: Flexible joint robots; lightweight robots; globally stable controller; passivity; vibration damping.

1. INTRODUCTION

The development of robotics in the past few years has gone from the earlier standard applications of industrial robots to new fields such as space and service robotics and force-feedback systems. A common feature that all robots suitable for these applications must share is a lightweight construction with a high load to weight ratio. The two lightweight robots at the DLR are designed with these considerations in mind [1, 2]. A main problem which is specific for the implementation of these new robot concepts is the inherent flexibility introduced into the robot joints. Consequently, the success in these robotics fields is strongly dependent on the design and implementation of adequate control strategies which, by making extensive use of sensory information, can compensate for the structural elasticity in the robot joints and provide fast control bandwidths near to those of industrial robots.

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In the past 15 years a large amount of research has been focused on the control of flexible joint robots. Starting from control methods developed for rigid-body manipulators, there are some very powerful theoretical results concerning the control of manipulators with joint elasticities. These include singular perturbation and integral manifold, feedback linearization, and dynamic feedback linearization along with adaptive control techniques. Integral manifold and singular perturbation techniques solve the control problem by a two-stage strategy [3, 4]. They propose a fast joint torque control loop, corresponding to the fast part of the manipulator dynamics, and a slower outer control loop, corresponding to the rigid-body dynamics of the robot. These control strategies use the assumption of a weak elasticity of the joints. In the case of the DLR lightweight robot this is just marginally satisfied. Our experiments [5] showed that the difficult part of this method is the implementation of the fast joint torque controller. Under conditions of considerable elasticity and noisy torque and torque derivative signals, the bandwidth of the resulting torque controller limits the overall bandwidth of the system. While implementation of force and impedance control showed good results, the position control proved to be slower than with other methods. The feedback linearization controller proposed by Spong [3] uses a somewhat simplified robot model. Even in this case the computations are much more involved compared to the equivalent computed torque method for rigid robots. In the most general case, the dynamics of the flexible joint robot is not feedback linearizable. De Luca [6] solves this problem by dynamic state linearization. He uses not only the actual state of the robot, but also the values of the past states, the resulting control structure having the order $2N(N-1)$. In order to overcome the main disadvantage of feedback linearization, i.e. the requirement of exact knowledge of robot parameters, adaptive control techniques have been proposed [7, 8].

Although these control methods are complete from the theoretical point of view, they are very difficult to implement. Mostly, tests are reported only through simulations or on one or two robot joints. For complex structures, such as the 7 d.o.f. redundant DLR robots, the involved computations required, the lack of robustness on parameter or model uncertainties and the difficulties in interpreting and debugging the results are serious obstacles when implementing these methods.

The practitioners attack the problem from the other end. One starts by implementing simple control structures such as PD controllers, which work for industrial robots, adding some damping for the joint elasticity or using linear techniques known from the control of elastic actuators. The bandwidth of the controllers has to be reduced until robustness against the highly non-linear dynamics is reached. Stability proofs for such controllers are more complicated than for controllers using extensive model information. Starting from [9], which provides the theoretical justification for the PD controller still used in most industrial robots, Tomei [10] proved the stability of PD control with gravity compensation also in the case of flexible joint robots. Since this controller uses only motor-side information, it is practically quite under-damped unless the bandwidth is considerably reduced.

This paper proposes an intermediate between the theoretical and the practical approaches presented before. The main focus of the paper is on a simple control structure in the form of a joint state feedback controller with gravity compensation. Since it uses both motor and link states, its performance is superior to that of the PD controller. In analogy to [9, 10] and a stability proof for this controller is presented, based on Lyapunov's first method. Compared to other controllers, this one is practically efficient and easy to implement, even for many d.o.f., and still theoretically well-founded. The control structure is then gradually extended by adding more information about the robot dynamics. This controller is presently used to control the new DLR robot and proved its efficiency in daily operation.

2. HARDWARE DESCRIPTION

The two lightweight robots developed at the DLR are very well suited for the implementation and testing of the control algorithms mentioned in the previous section. By designing highly integrated mechanical and electrical components, a load to own weight ratio of around 1 : 2 is achieved, for a robot weight of 17 kg. For joint control, the motor position and the joint torque signals based on strain gauge technology are available. The new robot (Fig. 1) is equipped with link position sensors as well. Additionally, a 6-d.o.f. force-torque sensor is integrated in the robot's wrist. Both robots are redundant 7-d.o.f. manipulators. The use of a floating

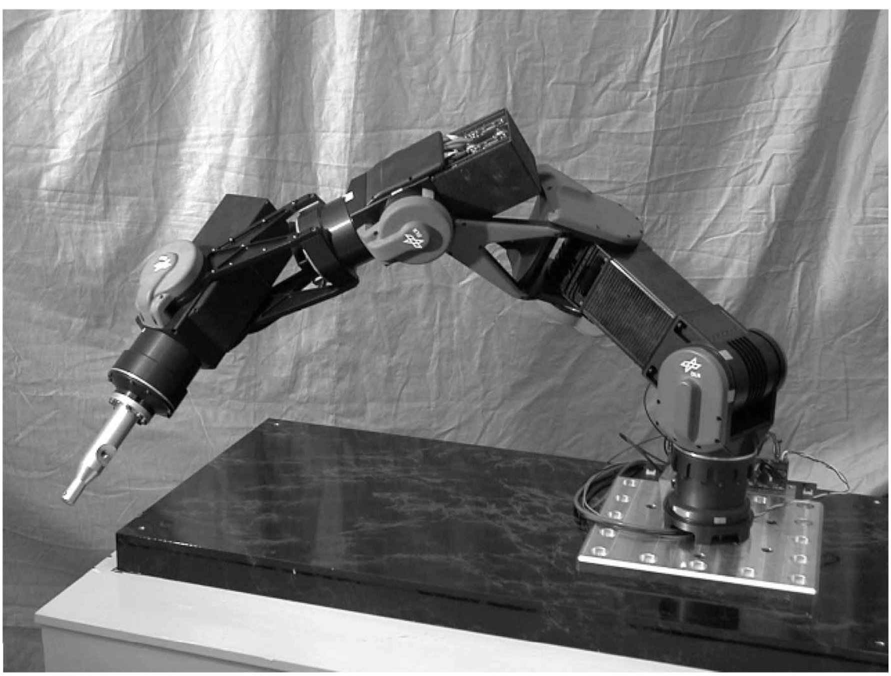

Figure 1. The new DLR lightweight robot.

point signal processor for each joint controller and a fast optical communication bus (1 ms) between the joints and the robot controller provides the necessary flexibility and computing power for involved control algorithms. The driving PM motors are controlled field oriented using analog Hall sensors, in order to reduce the motor torque ripple.

3. CONTROLLER STRUCTURE

For the design of the controller we start by considering the robot model proposed by Spong [3]:

$$\tau_{\mathrm{m}} = J\ddot{q}_1 + \tau + DK^{-1}\dot{\tau} + \tau_F, \tag{1}$$

$$\tau + DK^{-1}\dot{\tau} = M(q_2)\ddot{q}_2 + C(q_2, \dot{q}_2)\dot{q}_2 + g(q_2), \tag{2}$$

$$\tau = K(q_1 - q_2), \tag{3}$$

τ_{m} is the motor torque vector, q_1 and q_2 is the motor and link positions, respectively, and τ is the joint torque. J is the motor inertia matrix, and K and D are the elasticity and damping matrices, caused mainly by the gear box and the torque sensor. These matrices are diagonal and positive definite. M, C and g are the same as for stiff robots: the mass matrix, the Coriolis and centripetal torque vector, and the gravity vector. In these equations the kinetic energy of the rotors due to link movements is neglected, only the kinetic energy due to their own rotation being considered. For the reduction ratio of 160 (up to 606 in the first version) of our robots, this is a good approximation. τ_{F} is the friction force vector. For the simulation and compensation of friction we used the following model:

$$\begin{aligned}
&\text{if } |\dot{q}_1| \leqslant \varepsilon \\
&\quad \tau_{\mathrm{F}} = \tau_{\mathrm{m}} \quad \text{if } |\tau_{\mathrm{m}}| < \left|\tau_{\mathrm{F}}^{\max}\right|, \\
&\quad \tau_{\mathrm{F}} = \tau_{\mathrm{F}}^{\max} = \big(\tau_{\mathrm{C}} + \mu|\tau|\big)\operatorname{sign}(\tau_{\mathrm{m}}) \quad \text{else} \\
&\text{if } |\dot{q}_1| > \varepsilon \\
&\quad \tau_{\mathrm{F}} = \big(\tau_{\mathrm{C}} + \mu|\tau|\big)\operatorname{sign}(\dot{q}_1) + d_1\dot{q}_1,
\end{aligned} \tag{4}$$

τ_{C} and μ are the coefficients of the constant and the torque-dependent Coulomb friction, respectively. S and v_{s} parameterize the stiction and d_1 is the viscous friction coefficient. ε is zero in the real system and set to a small constant in simulation to avoid exact zero crossing detection.

The equations describing the dynamics of one joint can be split into a linear part and the non-linear terms, printed in boldface:

$$\begin{aligned}
\tau_{\mathrm{m}_i} &= J_i\ddot{q}_{1_i} + \tau_i + dk^{-1}\dot{\tau}_i + \boldsymbol{\tau}_{\mathbf{F_i}}, \\
\tau_i + dk^{-1}\dot{\tau}_i &= m_{ii}(q_{2_{i+1}}\dots q_{2_N})\ddot{q}_{2_i} \\
&\quad + \sum_{j\neq i}\mathbf{m_{ij}}(\mathbf{q_2})\ddot{\mathbf{q}}_{\mathbf{2_j}} + \mathbf{c_i}(\mathbf{q_2}, \dot{\mathbf{q}}_\mathbf{2})\dot{\mathbf{q}}_\mathbf{2} + \mathbf{g_i}(\mathbf{q_2}).
\end{aligned} \tag{5}$$

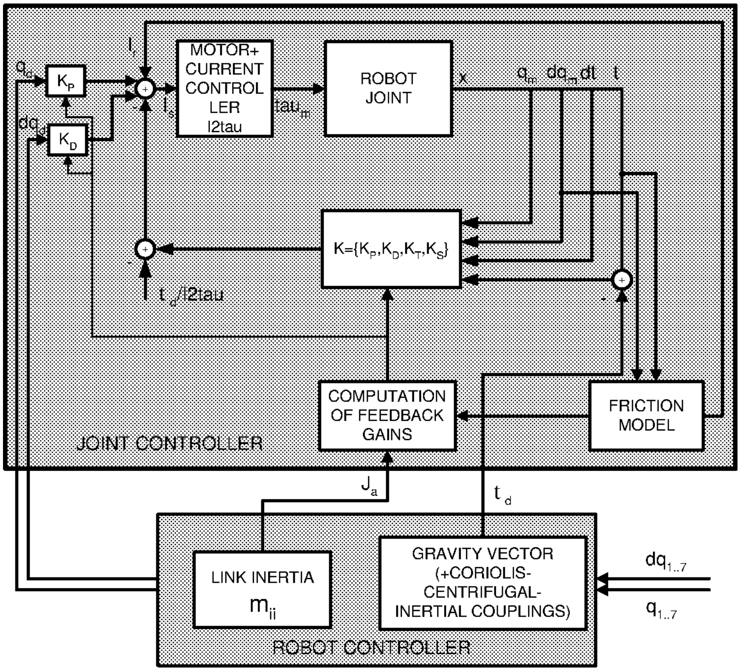

Figure 2. Controller structure.

Notice that the inertia m_{ii} is dependent only of the angles of the subsequent joints and not of the joint i itself.

The PD controller is a very simple and robust controller still used for most stiff industrial robots. Its straightforward extension to the case of flexible joint robots is a fourth-order state feedback controller for controlling the linear part in (5), while neglecting the non-linear terms. In subsequent stages, additional terms can complete the controller to compensate for gravity and friction, leading to the following control law:

$$\tau_{\mathrm{m}} = K_P\tilde{q}_1 - K_D\dot{q}_1 - K_T K^{-1}\tau - K_S K^{-1}\dot{\tau} + (K + K_T)K^{-1}g(q_{2\mathrm{d}}) + \tau_{\mathrm{F}}, \tag{6}$$

K_P, K_D, K_S and K_T are diagonal gain matrices. To compensate for the variation of m_{ii} or to implement variable joint stiffness and damping, the feedback gains can be modified online. As a last step, the centripetal and Coriolis terms, as well as inertial couplings can be compensated for. Figure 2 presents the proposed controller structure.

Six state signals are available for the joint control of the second robot, out of which three are obtained by direct measurement and the other three by numerical differentiation:

$$x_{\mathrm{m}} = \{q_2, \dot{q}_2, q_1, \dot{q}_1, \tau, \dot{\tau}\}. \tag{7}$$

This redundancy is very useful in practice. By fusion of redundant sensor information, the parameter identification is substantially simplified and improved. An advantage of using the torque signal in the control structure is that, by adequate setting of the feedback gains, it can be used to implement a torque controller or a stiff position controller. We used it to implement an impedance controller as well, which in fact covers the previous two structures as special cases.

The proposed controller structures can be regarded as successive simplifications of the feedback linearization controller, by omitting terms which are computationally expensive. With the available computing power, it turned out to be not possible to implement the complete feedback linearization controller in real time.

4. STABILITY PROOF FOR THE STATE FEEDBACK CONTROLLER WITH GRAVITY COMPENSATION

The Lyapunov proof makes use of some well known properties of the robot dynamic model [9, 10, 14]:

- (P1) Positive definiteness of the mass matrix

$$\lambda_{\mathrm{m}} \leqslant \left\| M(q_2) \right\| \leqslant \lambda_{\mathrm{M}}, \tag{8}$$

  with λ_{m}, λ_{M} the minimal and maximal eigenvalues of M.

- (P2) The matrix $\frac{1}{2}\dot{M}(q_2) - C(q_2, \dot{q}_2)$ is skew symmetric, hence:

$$\dot{q}_2^T \left[\frac{1}{2}\dot{M}(q_2) - C(q_2, \dot{q}_2) \right] \dot{q}_2 = 0. \tag{9}$$

- (P3) The gravity potential energy U_{G}, with:

$$\left(\frac{\partial U_{\mathrm{G}}(q_2)}{\partial q_{21}}, \ldots, \frac{\partial U_{\mathrm{G}}(q_2)}{\partial q_{2n}} \right) = g(q_2), \tag{10}$$

  is dominated by some quadratic function for a suitably chosen α:

$$\left| U_{\mathrm{G}}(q_{2\mathrm{d}}) - U_{\mathrm{G}}(q_2) + (q_2 - q_{2\mathrm{d}})^T g(q_{2\mathrm{d}}) \right| \leqslant \frac{1}{2}\alpha \|q_2 - q_{2\mathrm{d}}\|^2, \tag{11}$$

or, equivalently:

$$\|g(q_{2\mathrm{d}}) - g(q_2)\| \leqslant \alpha \|q_2 - q_{2\mathrm{d}}\|, \tag{12}$$

$q_{2\mathrm{d}}$ is the desired link position. We rewrite the dynamic equations only in terms of the motor position and velocity $q_1, \dot{q}_1$ and the link position and velocity $q_2, \dot{q}_2$.

$$\begin{aligned} K\Delta q + D\Delta\dot{q} &= M(q_2)\ddot{q}_2 + C(q_2, \dot{q}_2)\dot{q}_2 + g(q_2), \\ \tau_{\mathrm{m}} &= J\ddot{q}_1 + K\Delta q + D\Delta\dot{q} + \tau_{\mathrm{F}}. \end{aligned} \tag{13}$$

The control law (6) can be rewritten as:

$$\tau_{\mathrm{m}} = K_P\tilde{q}_1 - K_D\dot{q}_1 - K_T\Delta q - K_S\Delta\dot{q} + (K + K_T)K^{-1}g(q_{2\mathrm{d}}) + \tau_{\mathrm{F}}. \tag{14}$$

Here we use the following notations:

$$\Delta q = q_1 - q_2, \tag{15}$$

$$\tilde{q}_j = q_{j\mathrm{d}} - q_j, \quad j = 1, 2. \tag{16}$$

By setting all derivatives in (2) to zero for the desired equilibrium point, the reference motor position is related to the reference link position by:

$$q_{1\mathrm{d}} = q_{2\mathrm{d}} + K^{-1}g(q_{2\mathrm{d}}). \tag{17}$$

To prove the asymptotic stability of the controller, consider the following Lyapunov function candidate:

$$\begin{aligned} V = {} & \frac{1}{2}\dot{q}_1^T K(K + K_T)^{-1}J\dot{q}_1 + \frac{1}{2}\dot{q}_2^T M(q_2)\dot{q}_2 \\ & + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2)^T K(\tilde{q}_1 - \tilde{q}_2) + \frac{1}{2}\tilde{q}_1^T K_P K(K + K_T)^{-1}\tilde{q}_1 \\ & + U_{\mathrm{G}}(q_2) - U_{\mathrm{G}}(q_{2\mathrm{d}}) + \tilde{q}_2^T g(q_{2\mathrm{d}}). \end{aligned} \tag{18}$$

This function contains the kinetic energy of the motor and of the rigid body robot, the potential energy of the gravity force and of the link elasticity, and the energy corresponding to the controller. While for the proportional and the derivative terms the equivalence to a physical spring-damper suggests the choice of the energy, for the torque and torque derivative feedback such an analogy is not straightforward. A passivity-based method to find the energy function (18) of the state feedback controller is presented in Section 6.

If we use the state vector:

$$x = [\dot{q}_1, q_1, \dot{q}_1 - \dot{q}_2, q_1 - q_2], \tag{19}$$

and the reference vector:

$$x_{\mathrm{d}} = [0, q_{1\mathrm{d}}, 0, q_{1\mathrm{d}} - q_{2\mathrm{d}}], \tag{20}$$

then we have $V(x_{\mathrm{d}}) = 0$. To prove that $V(x) > 0$ for $x \neq x_{\mathrm{d}}$ we use property (P3) and obtain:

$$V \geqslant V_1 + V_2, \tag{21}$$

with:

$$V_1 = \frac{1}{2}\dot{q}_1^T K(K + K_T)^{-1}J\dot{q}_1 + \frac{1}{2}\dot{q}_2^T M(q_2)\dot{q}_2, \tag{22}$$

$$V_2 = \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2)^T K(\tilde{q}_1 - \tilde{q}_2) - \frac{1}{2}\tilde{q}_2^T \alpha \tilde{q}_2 + \frac{1}{2}\tilde{q}_1^T K_P K(K + K_T)^{-1}\tilde{q}_1. \tag{23}$$

For:

$$k_i + k_{Ti} > 0, \quad i = 1, \ldots, N, \tag{24}$$

and regarding (P1), V_1 is always positive. N is the number of robot joints. Since in V_2 all matrices are diagonal, V_2 can be written as:

$$V_2 = \sum_{i=1}^{N} V_{2i}, \tag{25}$$

V_{2i} are quadratic functions of $\tilde{q}_{1i}$ and $\tilde{q}_{2i}$ with the Hessian:

$$H(V_{2i}) = \frac{1}{2}\begin{bmatrix} k_i - \alpha & -k_i \\ -k_i & k_i\left(1 + \dfrac{k_{Pi}}{k_i + k_{Ti}}\right)\end{bmatrix}. \tag{26}$$

With (24), it follows that V_2 is p.d. if

$$\begin{cases} \alpha < k_i \\ \alpha < \dfrac{k_i k_{Pi}}{k_{Pi} + k_{Ti} + k_i}, \quad k_{Pi} > 0. \end{cases} \tag{27}$$

Then V is p.d. if the conditions (24) and (27) are satisfied. The derivative of the Lyapunov function V is:

$$\begin{aligned}\dot{V} &= \dot{q}_1^T K(K + K_T)^{-1} J\ddot{q}_1 + \dot{q}_2^T M(q_2)\ddot{q}_2 \\ &\quad + \frac{1}{2}\dot{q}_2^T \dot{M}(q_2)\dot{q}_2 - \Delta\dot{q}^T K(\tilde{q}_1 - \tilde{q}_2) \\ &\quad - \dot{q}_1^T K_P K(K + K_T)^{-1}\tilde{q}_1 + \dot{q}_2^T g(q_2) - \dot{q}_2^T g(q_{2\mathrm{d}}).\end{aligned} \tag{28}$$

In order to compute the derivative of V along the trajectories of the system we substitute (13), (14) into (28):

$$\begin{aligned}\dot{V} &= \dot{q}_1^T K(K + K_T)^{-1}\big[-(K + K_T)\Delta q + K_P\tilde{q}_1 \\ &\quad - K_D\dot{q}_1 - (D + K_S)\Delta\dot{q} + (K + K_T)K^{-1}g(q_{2\mathrm{d}})\big] \\ &\quad + \dot{q}_2^T\big[K\Delta q + D\Delta\dot{q} - C(q_2, \dot{q}_2)\dot{q}_2 - g(q_2)\big] \\ &\quad + \frac{1}{2}\dot{q}_2^T \dot{M}(q_2)\dot{q}_2 - \Delta\dot{q}^T K(\tilde{q}_1 - \tilde{q}_2) \\ &\quad - \dot{q}_1^T K_P K(K + K_T)^{-1}\tilde{q}_1 + \dot{q}_2^T\big(g(q_2) - g(q_{2\mathrm{d}})\big).\end{aligned} \tag{29}$$

By using (P2) we obtain after some simplifications:

$$\begin{aligned}\dot{V} = {} & -\dot{q}_1^T K(K+K_T)^{-1}\big[K_D\dot{q}_1 + (D+K_S)\Delta\dot{q}\big] \\ & + \dot{q}_2^T D\Delta\dot{q} - \Delta\dot{q}^T K\Delta q \\ & - \Delta\dot{q}^T K(\tilde{q}_1 - \tilde{q}_2) + \big(\dot{q}_1^T - \dot{q}_2^T\big)g(q_{2\mathrm{d}}). \end{aligned} \tag{30}$$

Using (17) and the fact, that

$$\tilde{q}_1 - \tilde{q}_2 = \Delta q_\mathrm{d} - \Delta q, \tag{31}$$

we finally get:

$$\begin{aligned}\dot{V} = {} & -\dot{q}_1^T K K_D(K+K_T)^{-1}\dot{q}_1 + \dot{q}_2^T D\Delta\dot{q} \\ & - \dot{q}_1^T K(D+K_S)(K+K_T)^{-1}\Delta\dot{q}. \end{aligned} \tag{32}$$

Since, again, all matrices are now diagonal, we have:

$$\dot{V} = \sum_{i=1}^{N} \dot{V}_i, \tag{33}$$

with

$$\begin{aligned}\dot{V}_i = {} & -\frac{1}{k+k_T}\big[(k_D + k_S + d)k\dot{q}_1^2 \\ & - \big((k_S + 2d)k + dk_T\big)\dot{q}_1\dot{q}_2 + (dk_T + dk)\dot{q}_2^2\big]. \end{aligned} \tag{34}$$

The indices i were omitted for the sake of simplicity. $\dot{V}$ is n.d. if the Hessian matrix $H(-\dot{V}_i)$ is p.d.:

$$H(-\dot{V}_i) = \frac{1}{2(k+k_T)}\begin{bmatrix} 2(k_D + k_S + d)k & -k_s k - 2dk - dk_T \\ -k_s k - 2dk - dk_T & 2(dk_T + dk) \end{bmatrix}. \tag{35}$$

With (24), this requires the following condition to be satisfied:

$$k_D > \frac{(k_S k - k_T d)^2}{4kd(k+k_T)}. \tag{36}$$

From (34) we see that $\dot{V}$ is just negative semidefinite. Thus we have to make use of Lasalle's invariance theorem [11, 12] to prove the asymptotic stability of the controlled robot. We have to prove that x_d is the maximal invariant set of the subspace:

$$x = \{\dot{q}_1 = 0, q_1, \dot{q}_2 = 0, q_2\}. \tag{37}$$

The condition (37) implies:

$$\begin{aligned} \ddot{q}_1 = 0, \quad \ddot{q}_2 = 0, \\ q_1 = \text{const.}, \quad q_2 = \text{const.} \end{aligned} \tag{38}$$

By imposing the conditions (38) on (13), (14) we obtain the equations of the largest invariant set.

$$K\,\Delta q - g(q_2) = 0,$$
$$-K\,\Delta q + K_P K (K + K_T)^{-1}\tilde{q}_1 + g(q_{2\mathrm{d}}) = 0, \tag{39}$$

using (17) for both $q_{1\mathrm{d}}$ and q_1, we get:

$$g(q_{2\mathrm{d}}) - g(q_2) = K K_P (K_P + K_T + K)^{-1}(q_2 - q_{2\mathrm{d}}). \tag{40}$$

But for $q_2 \neq q_{2\mathrm{d}}$ and considering (12) and (27) the following inequalities hold:

$$\begin{aligned}\|g(q_{2\mathrm{d}}) - g(q_2)\| &\leqslant \alpha \|(q_2 - q_{2\mathrm{d}})\| \\ &< \big\| K K_P (K_P + K_T + K)^{-1}(q_2 - q_{2\mathrm{d}})\big\|.\end{aligned} \tag{41}$$

So we can conclude that (40) cannot have other solutions than $q_2 = q_{2\mathrm{d}}$ and therefore the point $x = x_\mathrm{d}$ is globally asymptotically stable.

5. FRICTION INFLUENCE

Since friction is a dissipative force, the proof works also for the controller without or with inexact friction compensation. In this case V and (24), (27) remain unchanged. $\dot{V}$ contains the additional n.s.d. term $-\dot{q}_1^T K (K + K_T)^{-1}\tau_\mathrm{F}$ so that (36) is still valid. However, because of the Coulomb friction term, (40) changes to:

$$K^{-1}(K_P + K_T + K)\big(g(q_{2\mathrm{d}}) - g(q_2)\big) + K_P(q_{2\mathrm{d}} - q_2) = \tau_\mathrm{F}, \tag{42}$$

with $|\tau_\mathrm{F}| < \tau_\mathrm{F}^{\max}$ from (4). In this case, the maximal invariant set extends to an interval for each joint:

$$q_{2i} \in \big(q_{2\mathrm{d}i} - \tau_{\mathrm{F}i}^{\max}/\delta,\ q_{2\mathrm{d}i} + \tau_{\mathrm{F}i}^{\max}/\delta\big), \tag{43}$$

with:

$$\delta = k_P - \frac{\alpha(k_P + k_T + k)}{k}. \tag{44}$$

This reflects the fact that, with uncompensated Coulomb friction, the joint will stop within a dead zone around the desired position.

6. PASSIVITY CONSIDERATIONS ON THE JOINT CONTROLLER

In this section we use the passivity theory to provide a systematic way of deriving the Lyapunov function (18). We will show that the flexible joint robot, controlled with (6), can be regarded as a parallel and feedback connection of passive systems (Fig. 3), and hence it is itself a passive system.

Let us consider the actuator side equation of the robot dynamics (1) together with (3) and the controller (6) for each joint as a system with the input $u = -\dot{q}_2$

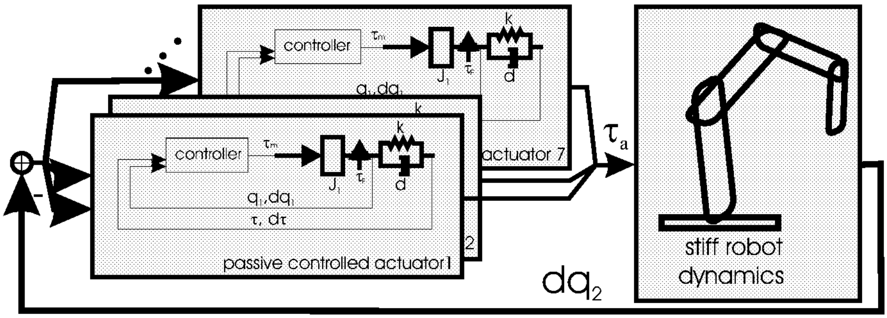

Figure 3. Representation of the robot as a connection of passive blocks.

and the output $y = \tau_a$, with $\tau_a = k(q_1 - q_2) + d(\dot{q}_1 - \dot{q}_2)$. We intend to write this system in the passivity form:

$$y^T(t)u(t) = \dot{V}(t) + D_p(t), \tag{45}$$

with the stored energy V, the dissipated power D_p and the external power exchange $y^T u$. We have:

$$\begin{aligned} -\tau_a\dot{q}_2 &= -k(q_1 - q_2)\dot{q}_2 - d(\dot{q}_1 - \dot{q}_2)\dot{q}_2 \\ &= \frac{d}{dt}\left(\frac{k}{2}(q_1 - q_2)^2\right) - k(q_1 - q_2)\dot{q}_1 - d(\dot{q}_1 - \dot{q}_2)\dot{q}_2. \end{aligned} \tag{46}$$

From (1), (3) and (6), by assuming exact friction compensation and ignoring the gravity, we get:

$$k(q_1 - q_2) = -\frac{k}{k + k_T}\big(J_1\ddot{q}_1 - k_P\tilde{q}_1 + k_D\dot{q}_1 + (d + k_S)(\dot{q}_1 - \dot{q}_2)\big). \tag{47}$$

Substituting (47) into (46) we finally obtain:

$$\begin{aligned} -\tau_a\dot{q}_2 = {} & \frac{d}{dt}\underbrace{\left(\frac{k_P k}{2(k + k_T)}\tilde{q}_1^2 + \frac{k}{2(k + k_T)}J_1\dot{q}_1^2 + \frac{1}{2}k(q_1 - q_2)^2\right)}_{V_i} \\ & + \underbrace{\frac{k_D k}{(k + k_T)}\dot{q}_1^2 - \frac{k_T d - k k_S}{k + k_T}(\dot{q}_1 - \dot{q}_2)\dot{q}_1 + d(\dot{q}_1 - \dot{q}_2)^2}_{D_{p_i}}. \end{aligned} \tag{48}$$

The energy expression V_i is always positive and D_{p_i} is a quadratic function of $\{\dot{q}_1, \dot{q}_2\}$, which can be rewritten as:

$$\begin{aligned} D_{p_i} = \frac{1}{k + k_T}\big(&(k_D + k_S + d)k\dot{q}_1^2 \\ &- \big((k_S + 2d)k + dk_T\big)\dot{q}_1\dot{q}_2 + (dk_T + dk)\dot{q}_2^2\big). \end{aligned} \tag{49}$$

It follows that the condition for the passivity of the system is $D_{\mathrm{p}_i} > 0$. This condition is equivalent to (36) and is the same as for the negative definiteness of $\dot{V}$ from (24).

The Lyapunov function for the whole robot (18) can be obtained as the sum of the energy V_i for each joint and the mechanical energy of the rigid robot

$$V_{\mathrm{rigid}} = \frac{1}{2}\dot{q}_2^T M(q_2)\dot{q}_2 + U_{\mathrm{G}}(q_2). \tag{50}$$

The last two terms in (18) are related to the gravity compensation term in (6), which is necessary for establishing the condition $V(q_{2\mathrm{d}}) = 0$, and hence for the global asymptotic stability proof.

An advantage of this controller structure over the PD control [10] is that the joint elasticity is included in the same passive block as the controller, providing the possibility to compensate for the effects of joint deformation. As it can be seen from the plots in Section 7, the controller provides an effective oscillation damping. It should be noticed that the controller structure (6) can implement by a proper parameterization a position, a stiffness or a torque controller while, as long as condition (36) is fulfilled, its passivity property is preserved.

7. EXPERIMENTAL RESULTS

A major practical step for the implementation of the proposed controller structure is the parameter identification. Because of the large number of parameters, we divided them in several groups which were identified separately. The robot's kinematic and dynamic parameters are very precisely computed using current mechanical CAD programs and measurement-based optimization of these parameters brought no considerable improvements. The friction parameters were identified based on current, torque and speed measurements on relevant trajectories for the whole robot. Although the characteristics of the current controlled motors can be identified together with the friction parameters, this leads to a bad conditioning of the optimization problem. Therefore the motor parameters were also identified separately using a motor testbed. The finite element method evaluation for the joint elasticity was not precise enough, so we determined it from the joint oscillation frequency, knowing the inertia. For the new robot, the available sensors enable online computation of the elasticity. The identification resulted in a very exact simulation model (Fig. 4), which is used for the design and test of the controllers. The values of the parameters for axis 2 of the second robot are listed in Table 1.

For a detailed description of the identification method and the results, as well as for further experimental data for all joints with the state feedback controller, please refer to [13].

Figures 5 and 6 present experimental comparisons between the proposed controller and a PD controller for both robots. One shows the case in which both

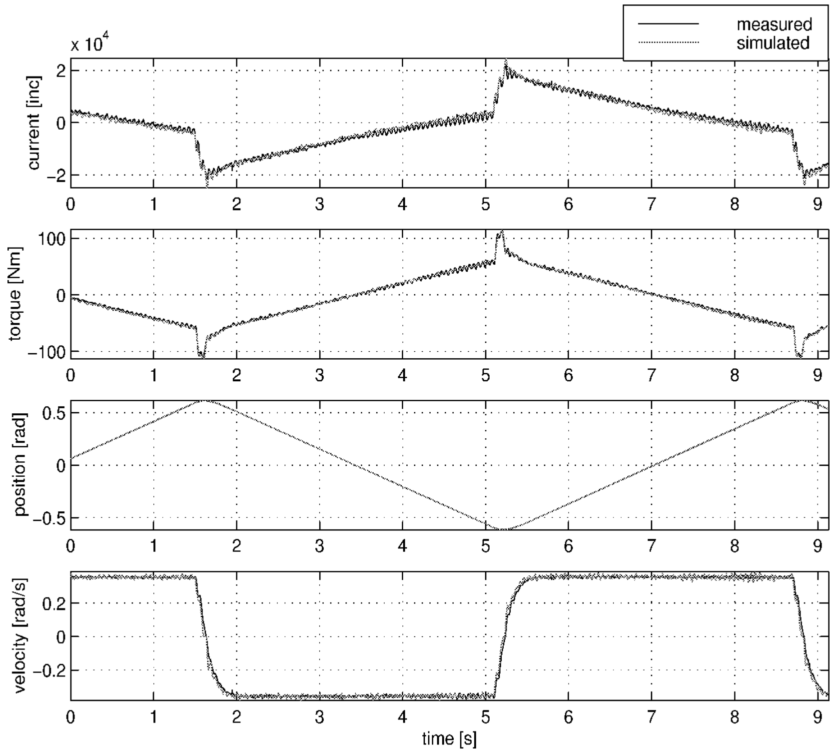

Figure 4. Parameter identification for axis 2 of the second DLR robot.

Table 1.

$J_{22} = 4.44$	$\mathrm{kg\,m^2}$
$m_{22\,\max} = 10.277$	$\mathrm{kg\,m^2}$
$g(q_2)_{\max} = 105.1$	$\mathrm{N\,m}$
$k = 1.4\ 10^4$	$\frac{\mathrm{N\,m}}{\mathrm{rad}}$
$d = 0.1075$	$\frac{\mathrm{s\,N\,m}}{\mathrm{rad}}$
$f_v = 41.9950$	$\frac{\mathrm{s\,N\,m}}{\mathrm{rad}}$

controllers have almost same bandwidth and the PD controller exhibits strong oscillations. The other one shows a better damped PD controller, which requires that the bandwidth is just half of the state feedback control bandwidth.

Figure 7 compares a state feedback controller with fixed gains with a variable gain controller on the first axis. During this experiment, the second axis was moved from 90° to 0°, so the link inertia for the first axis varied between $10.277 \div 0.1\ \mathrm{kg\,m^2}$. The fixed gain controller proves to work surprisingly well. By varying the gains, one can achieve higher bandwidth for low inertia.

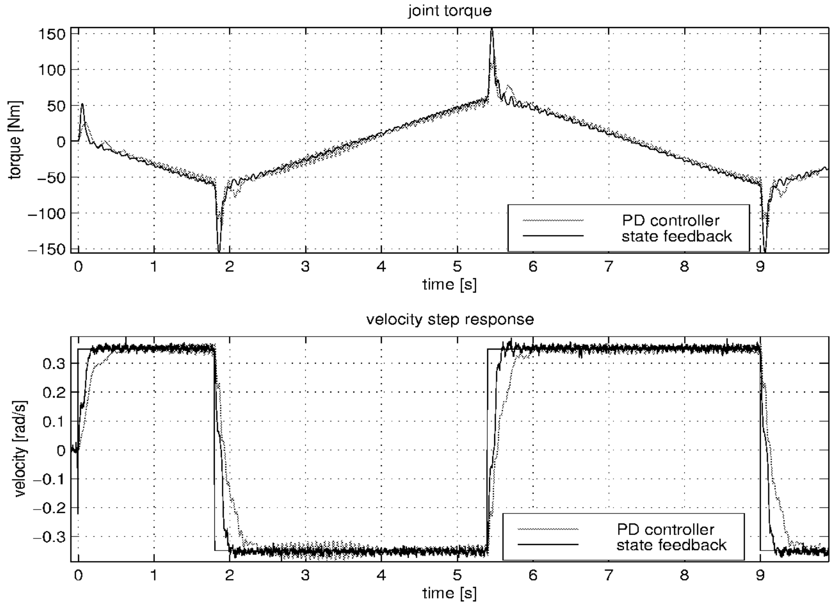

Figure 5. PD versus state feedback control on axis 2 of the second DLR robot.

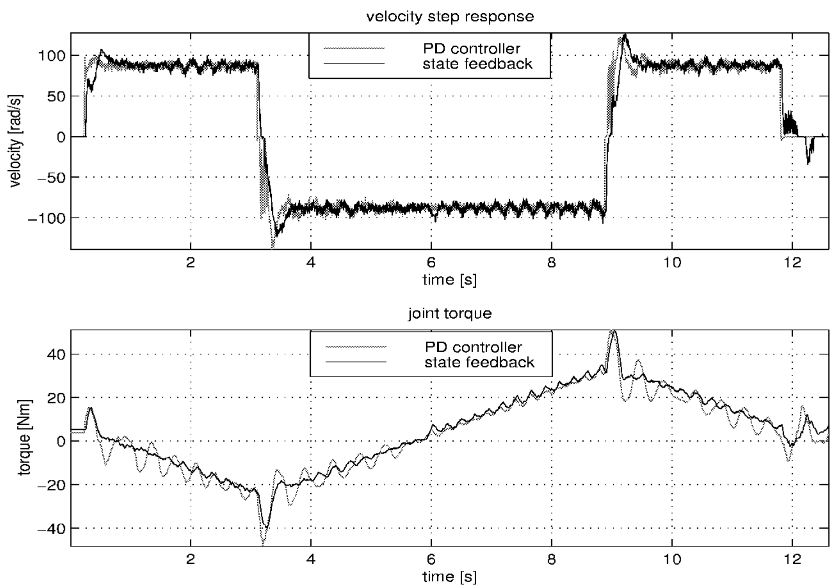

Figure 6. PD versus state feedback control on axis 2 of the first DLR robot.

8. SUMMARY

In this paper we proposed a state feedback controller for flexible joint robots which can be gradually extended to take account of the full robot dynamics. We proved that even with the simple, fixed gain controller, the arm can be stabilized

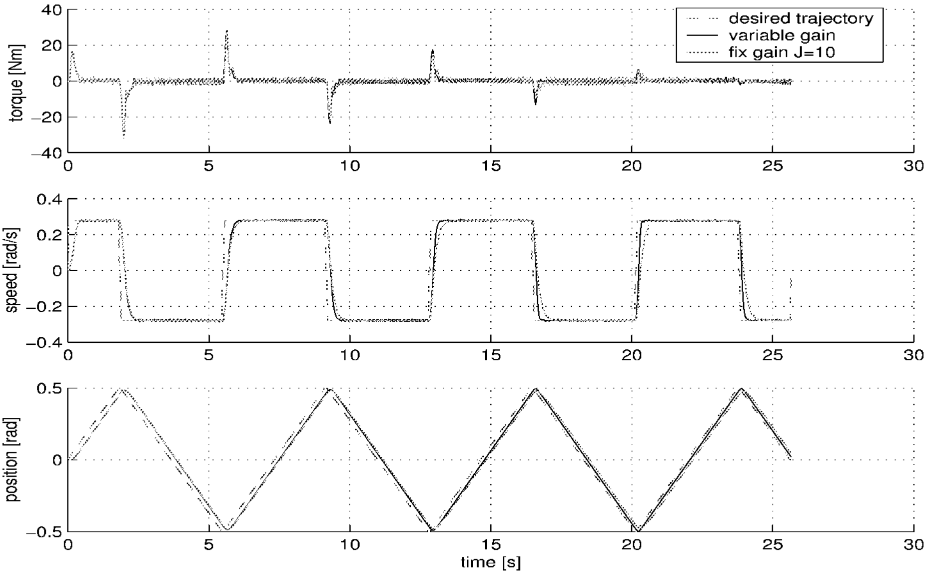

Figure 7. Fix gain and variable gain controller for axis 1. The inertia is continuously varying due to the movement of joint 2.

around a reference position and the oscillations caused by the joint flexibility are effectively damped. Compared to other controllers, this one is practically efficient and easy to implement even for many d.o.f., and still theoretically well-founded. The effectiveness of the controller is validated through experiments on the DLR lightweight robots.

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Since 1992 he has been director at DLR's Institute of Robotics and Mechatronics. He has published around 200 papers in robotics, mainly on robot sensing, sensory feedback, mechatronics, man-machine interfaces, telerobotics and space robotics. He was the first one who has sent a robot into space (shuttle mission ROTEX in April 93) and has controlled it from ground, too. In a large number of international robot conferences he was program committee member or invited plenary speaker. For many years he has been chairman of the German council on robot control and administrative committee member of the IEEE Society on Robotics and Automation.

He rejected several of chairs offered to him by different European Universities and received a number of high national and international awards, e.g. in 1994 the Joseph-Engelberger-Award for achievements in robotic science and in 1995 the Leibniz-Award, the highest scientific award in Germany and the JARA (Japan robotics association) Award. In 1996 he received the Karl-Heinz-Beckurts-Award, Germany's most important award for outstanding promotion of the partnership between science and industry, and in 1997 the IEEE-Fellow Award.