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Impedance and Interaction Control

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19.1 Introduction: Controlling Mechanical Interaction

Mechanical interaction with objects is arguably one of the fundamentally important robot behaviors. Many current robot applications require it; for example, mechanical interaction is essential for manipulation, the core task of assembly systems. Future robot applications, such as versatile use of tools or close cooperation with humans, may be enabled by improved control of mechanical interaction.

Interaction with the environment may serve sensory or motor functions (or both), and the most appropriate mechanical interaction is different for sensory or motor tasks. Mechanical interaction dynamics may be characterized by mechanical impedance, which may loosely be considered a dynamic extension of stiffness.[1] Lower mechanical impedance reduces interaction forces due to encountering an unpredicted object, thereby protecting both the robot and any object it manipulates (interaction forces on each being

[1] Conversely, mechanical admittance may be considered a dynamic generalization of compliance. A more rigorous definition is provided below.

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opposite but equal). Using a human analogy, by this reasoning, tactile exploration and manipulation of fragile objects should evoke the use of our lowest-impedance limb segments, and while we can (and routinely do) interact with objects using other body parts (the elbow, the knee, the foot, etc.), we naturally tend to use our fingers for gentle, delicate tasks.

Conversely, wielding an object as a tool often requires it to be stabilized and that requires higher mechanical impedance. This is especially important if the interaction between the manipulator and the object is destabilizing, as is the case for many common tools. Again using a human analogy, consider, for example, the simple task of pushing on a surface with a rigid stick. If force is exerted on the stick normal to the surface, then the stick is statically unstable; small displacements from the configuration in which stick axis and force vector co-align result in torques that act to drive the stick further from that configuration. Success at this task requires a stabilizing mechanical impedance, and because pushing harder exacerbates the problem (the magnitude of the destabilizing torque is proportional to the applied force), the minimum required impedance grows with the force applied; see Rancourt and Hogan (2001) for a detailed analysis. Simple though this task may be, it is an essential element of the function of many tools (e.g., screwdrivers, power drills) and any manipulator — human or robotic — must provide a stabilizing mechanical impedance to operate them.

In other applications the robot's interactive behavior may be the main objective of control. For example, to use a robot to serve as a force-reflecting haptic display (Miller et al., 2000) or to deliver physiotherapy (Volpe et al., 2000) requires intimate physical interaction with humans. In these applications the "feel" of the robot becomes an important performance measure, and "feel" is determined by mechanical interaction dynamics. Versatile interaction with objects (whether tools or humans or other robots), therefore, requires an ability to modulate and control the dynamics of interaction.

19.1.1 The Effect of Interaction on Performance and Stability

When interaction occurs, the dynamic properties of the environment are important. If we attempt to control motion or force, interaction affects the controlled variable, introducing error upon which the controller must act. Errors clearly mar performance but even more importantly, though perhaps less obvious, stability may also be compromised. That is, a system that is stable in isolation can become unstable when coupled to an environment that is itself stable. Such instabilities, known as coupled or contact instabilities, appear even in simple systems with basic controllers contacting simple environments. To illustrate this key point consider the following examples.

Example 19.1

The first example is based on a common design for a robot motion controller. One of the simplest models of a robot is depicted in Figure 19.1. A single mass *m* represents the robot's inertial properties. It is subject

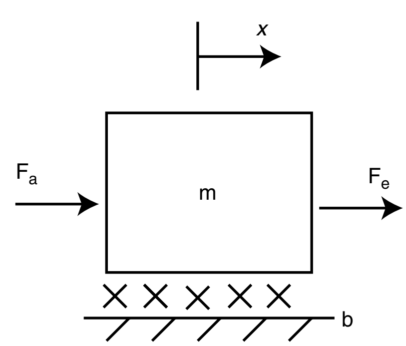


FIGURE 19.1 Model with inertial, frictional, actuator, and environmental forces.

to actuator forces F_a and environmental forces F_e. A single damper b connected to ground represents frictional losses. The Laplace-transformed equation of motion for this simple model is as follows:

$$(ms^2 + bs)X = F_a + F_e \tag{19.1}$$

where X is the Laplace transform of the mass position x. A proportional-integral motion controller is applied

$$F_a = K_p(R - X) + \frac{K_I}{s}(R - X) \tag{19.2}$$

where R is the Laplace transform of the reference position and K_p and K_I are proportional and integral gains, respectively. In isolation, $F_e = 0$ and the closed-loop transfer function is

$$\frac{X}{R} = \frac{K_P s + K_I}{ms^3 + bs^2 + K_P s + K_I} \tag{19.3}$$

From the Routh-Hurwitz stability criterion, a condition for *isolated* stability is the following upper bound on the integral gain:

$$K_I < \frac{bK_P}{m} \tag{19.4}$$

However, this condition is not sufficient to ensure that the robot will remain stable when it interacts with objects in its environment. Even the simple act of grasping an object may be destabilizing. If the system is not isolated, and instead is coupled to a mass m_e, this is equivalent to increasing the mass from m to $(m + m_e)$. Hence a condition for stability of the coupled system is

$$K_I < \frac{bK_P}{(m + m_e)} \tag{19.5}$$

Any controller that satisfies Equation (19.4) results in a stable isolated system. Prudent controller design would use a lower integral gain than the marginal value, providing robustness to uncertainty about system parameters and perhaps improved performance. However, for any fixed controller gains, coupling the robot to a sufficiently large m_e will violate the condition for stability. Interaction with a large enough mass can always destabilize a system that includes integral-action motion control, even if that system is stable in isolation.

This difficulty may be obscured by the facts that, at present, most robots have inertia far exceeding that of the payloads they may carry, the total mass $m + m_e$ is not much greater than the mass m of the robot alone, and the bounds of Equation (19.4) and Equation (19.5) are similar. However, in some applications, e.g., in space or under water, a robot need support little or none of a payload's weight, and objects of extremely large mass may be manipulated. In these situations the vulnerability of integral-action motion control to coupled instability may become an important consideration.

Example 19.2

The second example illustrates one of the common difficulties of force control. A robot with a single structural vibration mode is represented by the model shown in Figure 19.2. Two masses m_1 and m_2 are connected by a spring of stiffness k. Frictional losses are represented by dampers $b_1$and b_2 connected from the masses to ground and damper b_3 in parallel with the spring. One mass is driven by the actuator force F_a and the other is subject to F_e, an interaction force with the environment.

A proportional-derivative (PD) controller acting on the error between the position of the mass at the actuator x_1 and a reference r is applied to control motion. A proportional controller acting on force fed back from the environment is applied to control force and improve interactive behavior. The control law

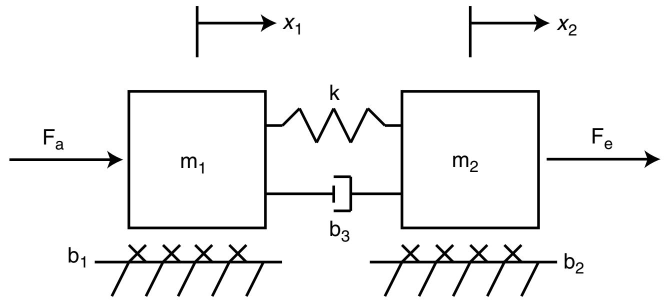


FIGURE 19.2 Model including a single structural resonance.

is as follows:

$$F_a = K(r - x_1) + B(\dot{r} - \dot{x}_1) + K_f(F_e + K(r - x_1) + B(\dot{r} - \dot{x}_1)) \tag{19.6}$$

where K and B are related to the proportional and derivative motion feedback gains, respectively, and K_f is the proportional force feedback gain.[2] When the system is isolated from any environment, $F_e = 0$ and Equation (19.6) reduces to PD motion control. Using the Routh-Hurwitz criterion, the isolated system's closed-loop characteristic polynomial can be shown to have all its poles in the open left half plane, providing at worst marginal stability for arbitrary nonnegative real parameters and controller gains.

If the system is not isolated but instead is connected to a spring to ground, such that $F_e = -k_e x_2$, the closed loop characteristic polynomial is changed. It is now easy to find parameters such that this polynomial has right-half-plane roots. For example, if $m_1 = m_2 = 10$, $b_1 = b_2 = b_3 = 1$, $k = 100$, $K = 10$, $B = 1$, $K_f = 10$, and $k_e = 100$, the closed loop poles are at $-2.78 \pm 5.51i$ and $2.03 \pm 5.53i$, where the latter pair are unstable. Once again, a system that is stable when isolated (corresponding to $k_e = 0$) is driven to instability when coupled to an object in its environment, in this case, a sufficiently stiff spring ($k_e = 100$).

Structural vibrations are a common feature of robot dynamics and present a substantial challenge to controller design. Though simple, the PD motion controller in this example has the merit that it is not destabilized by structural vibration because the actuator and assumed motion sensor are co-located.[3] However, the addition of a force feedback loop renders the robot control system vulnerable to coupled instability. In part this is because the actuator and force sensor are not co-located. Given the difficulty of designing a robot with no significant dynamics interposed between its actuators and the points at which it contacts its environment (see Tilley and Cannon, 1986, Sharon et al., 1988), the simple model in this example represents a common situation in robot interaction control, and we will return to this model several times in this chapter.

These two examples show that to ensure stability when interacting even with simple environments, it is not sufficient to design for stability of the manipulator in isolation. The environment's dynamics must also be considered — dynamics that are generally not known exactly. Furthermore, in many applications a robot must be capable of stable interaction with a variety of environments. For example, an assembly robot might pick up a component, move it across free space, bring it into contact with a kinematic constraint used to guide placement of the component, move it along the constraint, release the component, and return to get another. In this case, at different times in the process, the robot must remain stable when moving unloaded in free space, moving to transport the component, and moving to comply with the kinematic constraint. Each of these three contexts poses a different stability challenge which illustrates one

[2]This controller is discussed further below in the section on force feedback.
[3]An alternative interpretation is presented below in the section on simple impedance control.

key reason why, even if the environment can be closely modeled, there are strong benefits to designing a controller that is insensitive to ignorance of its properties. If, for example, a different controller is required for each of the three contexts, the system must decide which controller to use at each instant and manage the transition between them. Usually the easiest way to identify the environment is to interact with it; thus, interaction might be required before the appropriate controller is in place. Stable interaction is needed to identify the environment, but stability cannot be guaranteed without a well-characterized environment. A *single* controller that can perform satisfactorily within all expected contexts without compromising stability would have significant advantages.

19.1.2 Interaction as Disturbance Rejection

Control theory offers several tools to endow controllers with the ability to deal with unknown or poorly characterized interference (Levine, 1996). Using a disturbance rejection approach, the environment's dynamics could be included as disturbance forces. The success of this approach depends on bounding the disturbance forces, but for many interactive applications the environmental forces may equal or exceed the robot's nominal capacity; for example, a kinematic constraint can introduce arbitrarily large forces depending on the robot's own behavior. Furthermore, environmental forces generally depend on the robot's state, and traditionally disturbances are assumed to be state-independent. Thus, treating interaction as a disturbance rejection problem does not seem promising.

19.1.3 Interaction as Modeling Uncertainty

Modeling the environment as an uncertain part of the robot and using robust control tools (Levine, 1996) to guarantee stability is another reasonable approach. Interacting with an environment effectively changes the robot plant by adding some combination of elastic, dissipative, and inertial properties, perhaps including kinematic constraints. If the effect of interaction is only to alter the parameters of the robot model (e.g., by adding to the endpoint mass) a robust control approach may succeed, though robustifying the system to a large range of environment parameters might require an unacceptable sacrifice of performance. However, interaction may change the *structure* of the model. For example, it may reduce the order of the system (e.g., moving from free motion to contact with a kinematic constraint reduces the degrees of freedom of the parts contacting the constraint) or increase it (e.g., contact with an elastic object adds a new mode of behavior due to interaction between the robot inertia and the object elasticity). If interaction changes the model structure, the applicability of a robust control approach is unclear. For example, the environment forces and motions may be of the same magnitude and in the same frequency range as the known robot dynamics.[4] As robust control methods commonly assume that uncertain dynamics lie outside the frequency range of interest, treating interaction as a robustness problem does not seem promising.

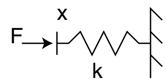
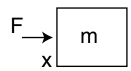
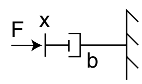
As mentioned above, in some applications the robot's dynamic behavior (colloquially termed its "feel") may be a key performance measure and hence a legitimate objective of control system design. Rather than attempt to overwhelm the consequences of interaction by building in robustness, the approach described herein aims to regulate the interaction itself by controlling the robot's dynamic behavior at the places where it interacts with its environment.

19.2 Regulating Dynamic Behavior

The general approach here termed "interaction control" refers to regulation of the robot's dynamic behavior at its *ports of interaction* with the environment. An interaction port is a place at which energy may be exchanged with the environment. It is therefore defined by a set of motion and force variables such that

[4]To use a human example, consider two people shaking hands: each has a similar response speed or bandwidth and produces comparable force and motion; each individual control system is coupled to a system with approximately double the large number of degrees of freedom that it controls when isolated.

TABLE 19.1 Laplace-Transformed Impedance and Admittance Functions for Common Mechanical Elements

	Impedance $F/\dot{x}$	Admittance $\dot{x}/F$
	$\frac{k}{s}$	$\frac{s}{k}$
	ms	$\frac{1}{ms}$
	b	$\frac{1}{b}$

$P = F^t v$, where v is a vector of velocities (or angular velocities) along different degrees of freedom at the contact point, F is a corresponding vector of forces (or torques), and P is the power that flows between robot and environment.[5] Interaction control involves specifying a dynamic relationship between motion and force at the port, and implementing a control law that attempts to minimize deviation from this relationship.

19.2.1 Mechanical Impedance and Admittance

The most common forms of interaction control regulate the manipulator's impedance or admittance. Assuming force is analogous to voltage and velocity is analogous to current, mechanical impedance is analogous to electrical impedance, characterized by conjugate variables that define power flow, and defined as follows.

Definition 19.1 Mechanical impedance at a port (denoted Z) is a dynamic operator that determines an output force (torque) time function from an input velocity (angular velocity) time function at the same port. Mechanical admittance at a port (denoted Y) is a dynamic operator that determines an output velocity (angular velocity) time function from a force (torque) time function at the same port.

The terms "driving point impedance" or "driving point admittance" are also commonly used. Because they are defined with reference to an interaction port, impedance and admittance are generically referred to as "port functions."

If the system is linear, admittance is the inverse of impedance, and both can be represented in the Laplace domain by transfer functions, $Z(s)$ or $Y(s)$. Table 19.1 gives example admittances and impedances for common mechanical elements. For a linear system impedance can be derived from the Laplace-transformed dynamic equations by solving for the appropriate variables.

While most often introduced for linear systems, impedance and admittance generalize to nonlinear systems. In the nonlinear case, using a state-determined representation of system dynamics, mechanical impedance may be described by state and output equations relating input velocity (angular velocity) to output force (torque) as follows.

[5]Equivalently, an interaction port may be defined in terms of "virtual work," such that $dW = F^t dx$ where dx is a vector of infinitesimal displacements (or angular displacements) along different degrees of freedom at the contact point, F is a corresponding vector of forces (or torques), and dW is the infinitesimal virtual work done on or by the robot.

$$\begin{aligned} \dot{z} &= Z_s(z, v) \\ F &= Z_o(z, v) \\ P &= F^t v \end{aligned} \tag{19.7}$$

where z is a finite-dimensional vector of state variables and $Z_s(\ )$ and $Z_o(\ )$ are algebraic (or memoryless) functions. The only difference from a general state-determined system model is that the input velocity and output force must have the same dimension and define by their inner product the power flow into (or out of) the system.

In the nonlinear case, mechanical admittance is the causal[6] dual of impedance in that the roles of input and output are exchanged; mechanical admittance may be described by state and output equations relating input force (torque) to output velocity (angular velocity). However, mechanical admittance may not be the inverse of mechanical impedance (and *vice versa*) as the required inverse may not be definable. Some examples of nonlinear impedances and admittances with no defined inverse forms are presented by Hogan (1985).

A key point to note is that, unlike motion or force, the dynamic port behavior is *exclusively a property of the robot system, independent of the environment* at that port. This is what gives regulating impedance or admittance its appeal. Motion and force depend on both robot and environment as they meet at an interaction port and cannot be described or predicted in the absence of a complete characterization of both systems. Indeed, this is the principal difficulty, as illustrated above, in regulating either of these quantities. Impedance, on the other hand, can (in theory) be held constant regardless of the environment; impedance defines the relationship between the power variables and does not by itself determine either.

Impedance serves to completely describe how the manipulator will interact with a variety of environments. In principle, if arbitrary impedance can be achieved, arbitrary behavior can be achieved; it remains only to sculpt the impedance to yield the desired behavior. Of course, as with all controller designs, the goal of achieving a desired impedance is an ideal; in practice, it can be difficult. The problem of achieving a desired impedance (or admittance) is central to the study of interaction control and is discussed in Section 19.4 and Section 19.5.

19.2.2 Port Behavior and Transfer Functions

Port impedance and admittance are just two of the possible representations of a linear system's response, and it is important to highlight the distinction between these expressions and general input/output transfer functions. By way of example, consider again the system in Figure 19.2. This system is an example of a "2-port" because it has two power interfaces, one characterized by F_a and $\dot{x}_1$, the other by F_e and $\dot{x}_2$. If such an element is part of a robot, one side is generally connected to an actuator and the other to a downstream portion of the robot or directly to the environment. Any mechanical 2-port has four transfer functions relating motion to force. Two of them, $\frac{F_a}{\dot{X}_2}(s)$ and $\frac{F_e}{\dot{X}_1}(s)$ (or their inverses), are input/output transfer functions and define the force produced by motion at the opposite port, assuming certain boundary conditions to determine the other power variables. This type of transfer function is used in traditional block-diagram analysis; if the left port is driven by a controllable force and the right uncoupled, then $\frac{\dot{X}_2}{F_a}(s)$ describes the motion of the right port as a result of actuator force, as shown in Figure 19.3. The other two transfer functions, $\frac{F_a}{\dot{X}_1}(s)$ and $\frac{F_e}{\dot{X}_2}(s)$, each represent the impedance at a port (and their inverses the corresponding admittance), depending on the boundary conditions at the other port.

The principal distinction between these two types of transfer functions is in their connection. If we have two systems like that shown in Figure 19.2, their input/output transfer functions cannot be properly

[6]The causal form (or *causality*) of a system's interaction behavior refers to the choice of input and output variables. By convention, input variables are referred to as "causing" output variables, though the physical system equations need not describe a causal relation in the usual sense of a temporal order; changes of input and output variables may occur simultaneously.

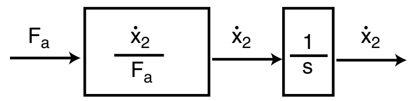


FIGURE 19.3 Block diagram of a forward-path or transmission transfer function.

connected in cascade because the dynamics of the second system affect the input-output relationship of the first. At best, cascade combination of input/output transfer functions may be used as an approximation if the two systems satisfy certain conditions, specifically that the impedance (or admittance) at the common interaction port are maximally mismatched to ensure no significant loading of one system by the other.

19.3 Analyzing Coupled Systems

To analyze systems that are coupled by interaction ports, it is helpful to consider the causality of their interaction behavior, that is, the choice of input and output for each of the systems. The appropriate causality for each system is constrained by the nature of the connection between systems, and in turn affects the mathematical representation of the function that describes the system. Power-based network modeling approaches (such as bond graphs [Brown, 2001]) are useful, though not essential, to understand this important topic.

19.3.1 Causal Analysis of Interaction Port Connection

Figure 19.4 uses bond graph notation to depict direct connection of two mechanical systems, S_1 and S_2, such that their interaction ports have the same velocity (the most common connection in robotic systems), denoted by the **1** in the figure. If we choose to describe the interaction dynamics of one system as an impedance, causal analysis dictates that the other system must have admittance causality. The causal stroke indicates that S_1 is in impedance causality; in other words, S_1 takes motion as an input and produces force output, and S_2 takes force as an input and produces motion output. If we use the common sign convention that power flow is positive into each system (denoted by the half arrows in the figure) this is equivalent to a negative feedback connection of the two power ports, as shown in Figure 19.5.

More complicated connections are also possible. Consider three mechanical systems connected such that their common interaction ports have the same velocity (e.g., consider two robots S_3 and S_4 pushing

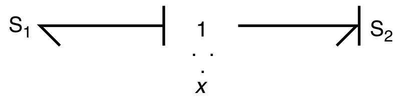


FIGURE 19.4 Bond graph representation of two systems connected with common velocity.

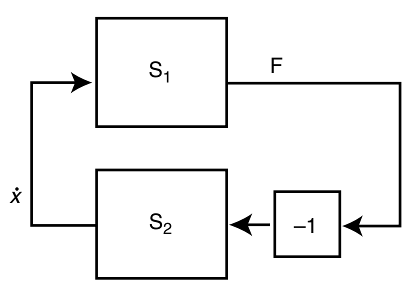


FIGURE 19.5 Feedback representation of two systems connected with common velocity.

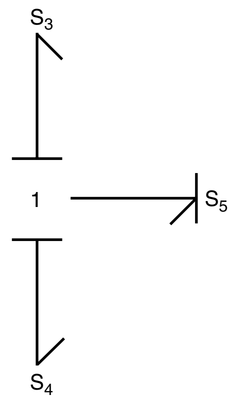


FIGURE 19.6 Bond graph representation of three systems connected with common velocity.

on the same workpiece S_5). Figure 19.6 shows a bond graph representation. If we choose to describe the workpiece dynamics as an admittance (and this may be the only option if the workpiece includes a kinematic constraint) then causal analysis dictates that the interaction dynamics of the two robots must be represented as impedances. Once again, this connection may be represented by feedback network. S_3 and S_4 are connected in parallel with each other and with the workpiece S_5 in a negative feedback loop, as shown in Figure 19.7. Note that S_3 and S_4 can be represented by a single equivalent impedance, and the system looks like that in Figure 19.4 and Figure 19.5. S_3 and S_4 need not represent distinct pieces of hardware and, in fact, might represent superimposed control algorithms acting on a single robot (Andrews and Hogan, 1983; Hogan, 1985).

19.3.2 Impedance vs. Admittance Regulation

Causal analysis provides insight into the important question whether it is better to regulate impedance or admittance. For most robotics applications, the environment consists primarily of movable objects, most simply represented by their inertia and surfaces or other mechanical structures that kinematically constrain their motion. The interaction dynamics in both cases may be represented as an admittance. An unrestrained inertia determines acceleration in response to applied force, yielding a proper admittance transfer function (see Table 19.1). A kinematic constraint imposes zero motion in one or more directions regardless of applied force; it does not admit representation as an impedance. Inertias prefer admittance causality; kinematic constraints require it. Because the environment has the properties best represented

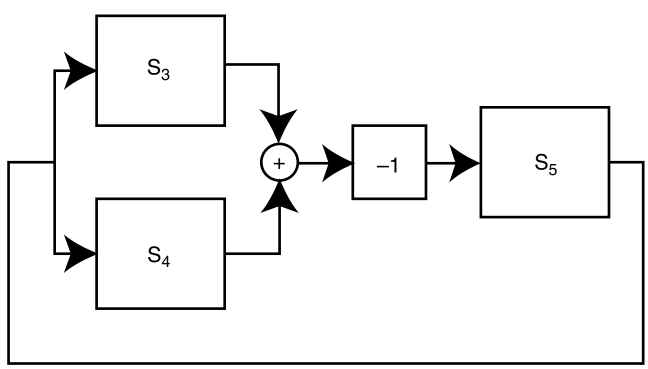


FIGURE 19.7 Feedback representation of three systems connected with common velocity.

as an admittance, the ideal robot behavior is an impedance, which can be thought of as a dynamic generalization of a spring, returning force in response to applied displacement. Initially assuming that arbitrary port impedance could be achieved, Hogan argued for this approach (Hogan, 1985).

However, most robots consist of links of relatively large mass driven by actuators. If the actuators are geared to amplify motor torque (a common design), the total inertia apparent at the end effector is increased by the reflected inertia of the motor; indeed for high gear ratios this can dwarf the mass of the links. These inertial properties are difficult to overcome (see Section 19.5.2), and tend to dominate the robot's response. Thus it is difficult to make a robot behave as a spring (or impedance), and it is usually more feasible to make it behave as primarily a mass (or admittance), an argument for admittance control (Newman, 1992). In other words, impedance behavior is usually ideal, but admittance behavior is often more easily implemented in real hardware, which itself prefers admittance causality because of its inertial nature. The choice of particular controller structure for an application must be based on the anticipated structure of environment and manipulator, as well as the way that they are coupled.

19.3.3 Coupled System Stability Analysis

If S_1 and S_2 are a linear time-invariant impedance and admittance, respectively, Figure 19.4 and Figure 19.5 depict the junction of the two systems via a power-continuous coupling. In other words the coupling is lossless and does not require power input to be implemented; all power out of one system goes into the other. Colgate has demonstrated the use of classical feedback analysis tools to evaluate the stability of this coupled system, using only the impedance and admittance transfer functions (Colgate, 1988; Colgate and Hogan, 1988). By interpreting the interaction as a unity negative feedback as shown in Figure 19.5, Bode and/or Nyquist frequency response analysis can be applied. The "open-loop" transfer function in this case is the product of admittance and impedance $S_1 S_2$. Note that unlike the typical cascade of transfer functions, this product is exact, independent of whether or how the two systems exchange power or "load" each other. The Nyquist stability criterion ensures stability of the coupled (closed-loop) system if the net number of clockwise encirclements of the -1 point (where magnitude $= 1$ and phase angle $= \pm 180°$) by the Nyquist contour of $S_1(j\omega)S_2(j\omega)$ plus the number of poles of $S_1(j\omega)S_2(j\omega)$ in the right half-plane is zero. The slightly weaker Bode condition ensures stability if the magnitude $|S_1(j\omega)S_2(j\omega)| < 1$ at any point where $\angle(S_1(j\omega)S_2(j\omega) = \angle S_1(j\omega) + \angle S_2(j\omega) = \pm 180°$.

19.3.4 Passivity and Coupled Stability

This stability analysis requires an accurate representation of the environment as well as the manipulator, something that we have argued is difficult to obtain and undesirable to rely upon. The principles of this analysis, however, suggest an approach to guarantee stability if the environment has certain properties — particularly if the environment is passive.

There are a number of definitions of a system with passive port impedance, all of which quantify the notion that the system cannot, for any time period, output more energy at its port of interaction than has in total been put into the same port for all time. Linear and nonlinear systems can have passive interaction ports; here we restrict ourselves to the linear time-invariant case and use the same definition as Colgate (1988). For nonlinear extensions, see Wyatt et al. (1981) and Willems (1972).

Definition 19.2 A system defined by the linear 1-port[7] impedance function $Z(s)$ is passive iff:

1. $Z(s)$ has no poles in the right half-plane.
2. Any imaginary poles of $Z(s)$ are simple, and have positive real residues.
3. $\text{Re}\{Z(j\omega)\} \geq 0$.

[7] An extension to multi-port impedances and admittances is presented in Colgate (1988).

These requirements ensure that $Z(s)$ is a positive real function and lead to the following interesting and useful extensions:

1. If $Z(s)$is positive real, so is its inverse, the admittance function $Y(s) = Z^{-1}(s)$, and $Y(s)$ has the same properties.
2. If equality is restricted from condition 3, the system is dissipative and is called *strictly passive*.
   a. The Nyquist contours of $Z(s)$ and $Y(s)$ each lie wholly within the closed right half-plane.
   b. $Z(s)$ and $Y(s)$ each have phase in the closed interval between -90° and $+90^\circ$.
3. If condition 3 is met in equality, the system is passive (but not strictly passive).
   a. The Nyquist contours of $Z(s)$ and $Y(s)$ each lie wholly within the open right half-plane.
   b. $Z(s)$ and $Y(s)$ each have phase in the open interval between -90° and $+90^\circ$.

Note that pure masses and springs, as shown in Table 19.1, are passive but not strictly passive. Pure dampers are strictly passive, as they have zero phase. Furthermore, any collection of passive elements (springs, masses, dampers, constraints) assembled with any combination of power-continuous connections is also passive. Passive systems comprise a broad and useful set, including all combinations of mechanical elements that either store or dissipate energy without generating any — even when they are nonlinear. Passivity and related concepts have proven useful for other control system applications, including robust and adaptive control (Levine, 1996).

Colgate has shown that the requirement for a manipulator to interact stably with any passive environment is that the manipulator itself be passive (Colgate, 1988; Colgate and Hogan, 1988). The proof is described here intuitively and informally. If two passive systems are coupled in the power-continuous way described above and illustrated in Figure 19.4, the phase property of passive systems constrains the total phase of the "open-loop" transfer function to between -180° and $+180^\circ$ (the phase of each of the two port functions is between -90° and $+90^\circ$, and the two are summed). Because the phase never crosses these bounds, the coupled system is never unstable and is at worst marginally stable. This holds true *regardless of the magnitudes* of the port functions of both systems. In the Nyquist plane, as shown in Figure 19.8, since the total phase never exceeds 180°, the contour cannot cross the negative real axis, and therefore can never encircle -1, regardless of its magnitude. This result shows that if a manipulator can be made to behave with passive driving point impedance, coupled stability is ensured with *all* passive environments, provided the coupling obeys the constraints applied above. The magnitude of the port functions is irrelevant; if passivity (and therefore the phase constraint) is satisfied, the coupled system is stable. The indifference to magnitude also means that requiring a robot to exhibit passive interaction behavior need not compromise performance; in principle the system may be infinitely stiff or infinitely compliant.

It is worthy of mention that if either of the two systems is *strictly* passive, the total phase is strictly less than $\pm 180^\circ$, and the coupled system is asymptotically stable. Different types of coupling can produce slightly different results. If the act of coupling requires energy to be stored or if contact is through a mechanism such as sliding friction, local asymptotic stability may not be guaranteed. However, the coupled system energy remains bounded. This result follows from the fact that neither system can generate energy or supply it continuously; the two can only pass it back and forth and the total energy can never grow.

Example 19.3

Because both examples presented at the start of this paper were unstable when interacting with passive elements, both systems must be nonpassive. This is in fact true; for the second case, for example, Figure 19.9 shows the real part of the port admittance, evaluated as a function of frequency, and it is clearly negative between 2 and 10 rad/sec, hence violates the third condition for passivity.

Colgate has proven another useful result via this argument, particularly helpful in testing for coupled stability of systems. As can easily be determined from Table 19.1, an ideal spring in admittance causality produces $+90^\circ$ of phase, and an ideal mass produces -90°, both for all frequencies, making each passive

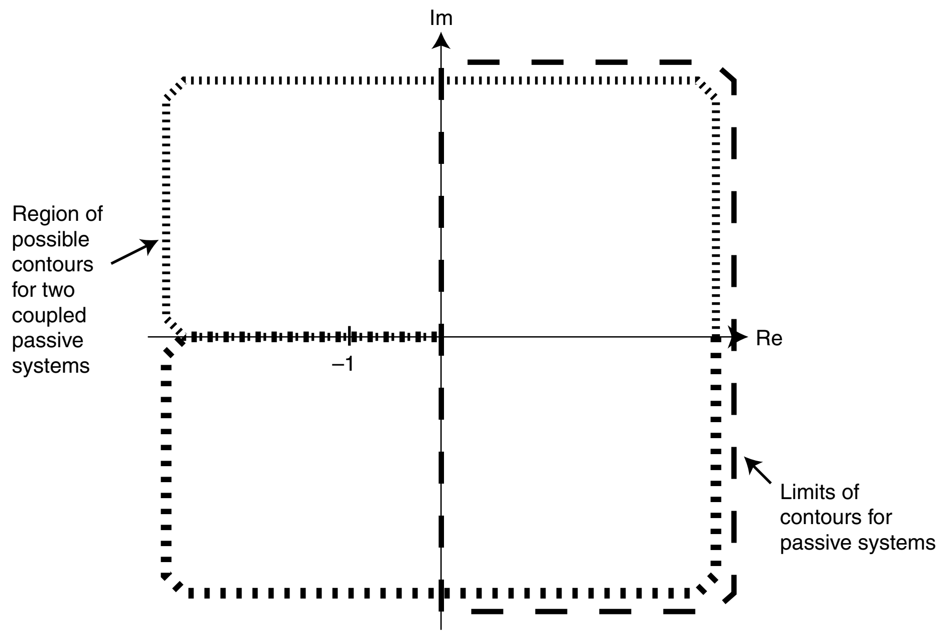


FIGURE 19.8 Region of the Nyquist plane that contains contours for passive systems (dashed), and region that can contain coupled passive systems (dotted). The coupled Nyquist contour may lie on the negative real axis but cannot encircle the −1 point. If a system is *strictly* passive, its Nyquist plot cannot lie on the imaginary axis. If at least one of two coupled passive systems is *strictly* passive, then its shared Nyquist contour cannot intersect or lie on the negative real axis.

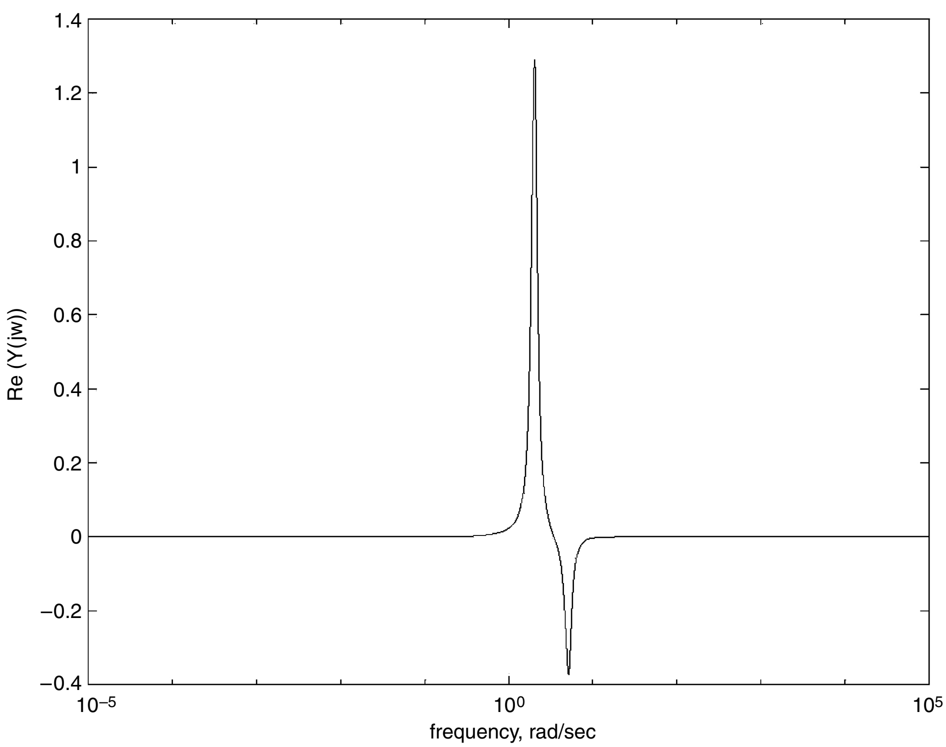


FIGURE 19.9 Real part of the interaction port admittance of the system from Example 2.

but not strictly passive. If the manipulator is nonpassive, its phase must necessarily exceed either −90° or +90° at some frequency, so that coupling it to a pure spring or a pure mass results in phase that exceeds −180° or +180° at that frequency. The value of the environmental stiffness or mass acts as a gain and can be selected such that the magnitude exceeds 1 at a frequency where the phase crosses the boundary, proving instability by the Bode criterion. Alternatively by the Nyquist criterion, the gain expands or contracts the contour, and can be selected so as to produce an encirclement of the −1 point. Conversely, it is impossible for the manipulator to be unstable when in contact with any passive environment if it does not become unstable when coupled to any possible spring or mass. Thus the passivity of a manipulator can theoretically be evaluated by testing stability when coupled to all possible springs and masses. If no spring or mass destabilizes the manipulator, it is passive. Much can be learned about a system by understanding which environments destabilize it (Colgate, 1988; Colgate and Hogan, 1988).

Example 19.4

The destabilizing effect of springs or masses can be observed by examining the locus of a coupled system's poles as an environmental spring or mass is changed. Figure 19.10 shows the poles of a single-resonance system (as in Example 19.2) coupled to a mass at the interaction port, as the magnitude of the mass is varied from 0.1 to 10 kg. The system remains stable. Figure 19.11 shows the coupled system poles as a stiffness connected to the interaction port is varied from 1 to 150 N/m. When the stiffness is large, the system is destabilized.

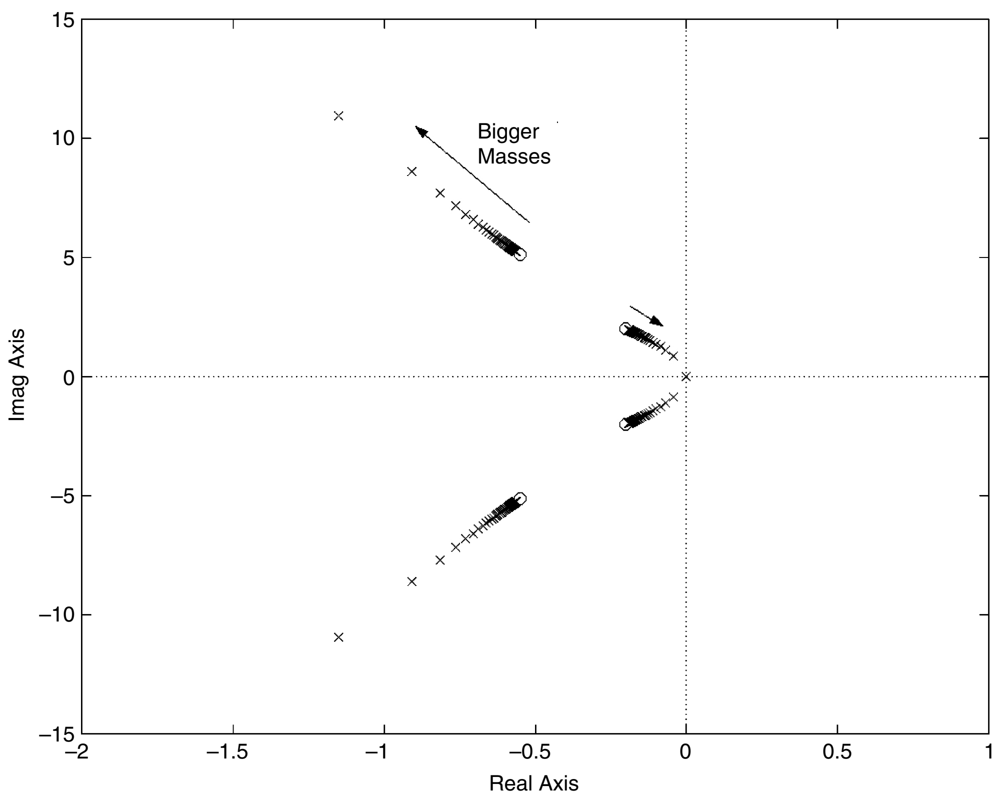


FIGURE 19.10 Locus of coupled system poles for the system from Example 19.2 as environment mass increases.

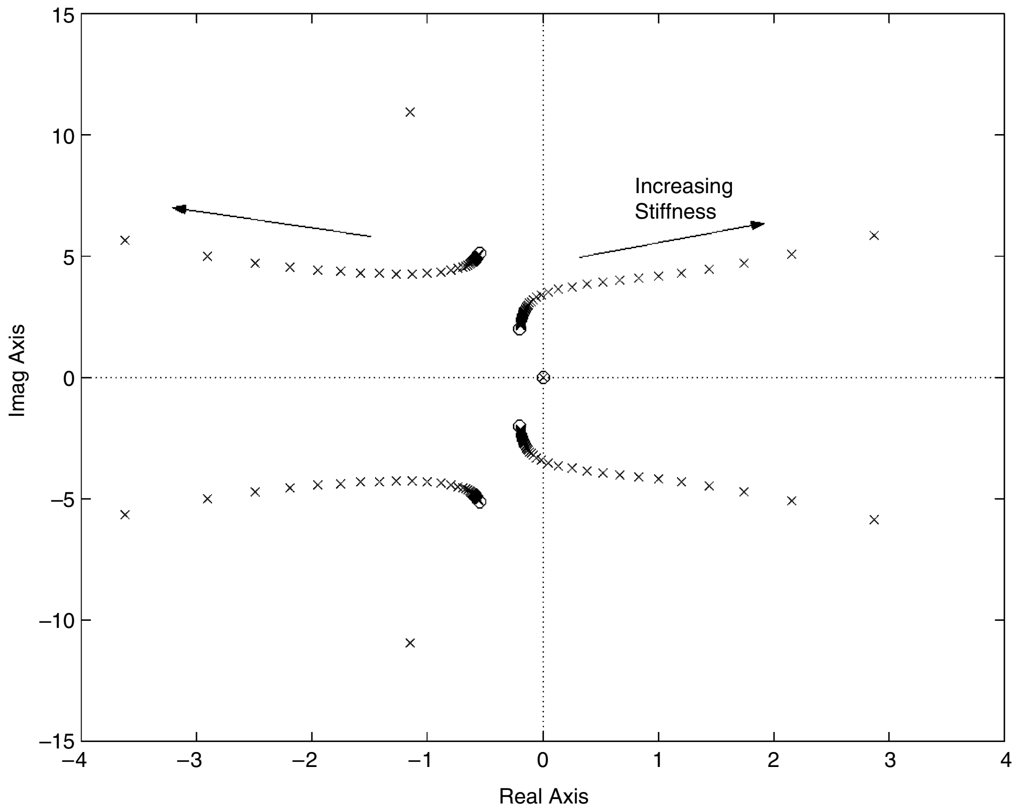


FIGURE 19.11 Locus of coupled system poles for the system from Example 19.2 as environment stiffness increases.

19.4 Implementing Interaction Control

Like any control system, a successful interactive system must satisfy the twin goals of stability and performance. The preceding sections have shown that stability analysis must necessarily include a consideration of the environments with which the system will interact. It has further been shown that stability can in principle be assured by sculpting the port behavior of the system. Performance for interactive systems is also measured by dynamic port behavior, so both objectives can be satisfied in tandem by a controller that minimizes error in the implementation of some target interactive behavior. For this approach to work for manipulation, any controller must also include a way to provide a motion trajectory. The objectives of dynamic behavior and motion can, to some degree, be considered separately.

19.4.1 Virtual Trajectory and Nodic Impedance

Motion is typically accomplished using a virtual trajectory, a reference trajectory that specifies the desired movement of the manipulator. A virtual trajectory is much like a motion controller's nominal trajectory, except there is no assumption that the controlled machine's dynamics are fast in comparison with the motion, which is usually required to ensure good tracking performance. The virtual trajectory specifies which way the manipulator will "push" or "pull" to go, but actual motion depends on its impedance properties and those of the environment. It is called "virtual" because it does not have to be a physically realizable trajectory.

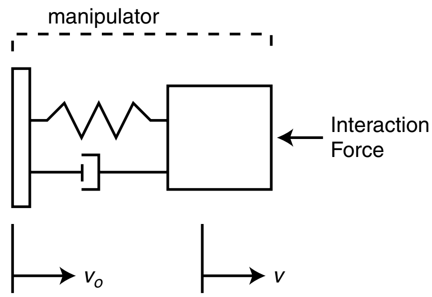


FIGURE 19.12 Virtual trajectory v_o and nodic impedance. v is the interaction port velocity.

Deviation in the manipulator's trajectory from its virtual trajectory produces a force response that depends on the interactive properties of the manipulator, its *nodic*[8] impedance/admittance. If the robot is represented as a mass subjected to actuator and environmental forces (similar to the simple model of Example 19.1 above) and the nodic impedance of the controller is a parallel spring and damper, the resulting behavior is as pictured in Figure 19.12 in which v_o represents the virtual trajectory and v the actual manipulator motion. In many applications, the controller will be required to specify manipulator position x and to be consistent, the virtual trajectory must also be specified as a position x_o. In that case, the noninertial (nodic) behavior is strictly described as a dynamic operator that produces output force in response to input displacement (the difference between virtual and actual positions, $\Delta x = x_o - x$), which might be termed a "dynamic stiffness." However, the term "impedance" is loosely applied to describe either velocity or displacement input. To the interaction port, the system behaves as a mass with a spring and damper, connecting the port to some potentially moving virtual trajectory. The dynamics of the spring, mass, and dashpot are as much a part of the prescribed behavior as is the virtual trajectory, distinguishing this strategy from motion control.

19.4.2 "Simple" Impedance Control

One primitive approach to implementing impedance control, proposed by Hogan (1985), has been used with considerable success. Termed "simple" impedance control, it consists of driving an intrinsically low-friction mechanism with force- or torque-controlled actuators, and using motion feedback to increase output impedance. This approach makes no attempt to compensate for any physical impedance (mass, friction) in the mechanism, so the actual output impedance consists of that due to the controller plus that due to the mechanism.

If a robot is modeled as a multi-degree-of-freedom inertia retarded by damping and subject to actuator and environmental torques (a multi-degree-of-freedom version of the model used in example 1 above), the robot with the simple impedance controller is as follows:

$$I(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta}) + D(\dot{\Theta}) = T_a + T_e \tag{19.8}$$

where Θ is a vector of robot joint variables (here assumed to be angles, though that is not essential), I is the robot inertia matrix (which depends on the robot's pose), C denotes nonlinear inertial coupling torques (due to Coriolis and/or centrifugal accelerations), D is a vector of dissipative velocity-dependent torques (e.g., due to friction), T_a is a vector of actuator torques, and T_e a vector of environmental torques. The target behavior is impedance; if the behavior of a spring with stiffness matrix K_j and a damper with

[8]The term *nodic* refers to "transportable" behavior that may be defined relative to a nonstationary (or even accelerating) reference frame. Nodicity is discussed by Hogan (1985) and Won and Hogan (1996).

damping matrix B_j is chosen, the control law is simply

$$T_a(\Theta, \dot{\Theta}) = K_j(\Theta_o - \Theta) + B_j(\dot{\Theta}_o - \dot{\Theta}) \tag{19.9}$$

where Θ_o is a virtual trajectory in robot joint space. Combining the controller impedance (Equation 19.9) with the robot dynamics (Equation 19.8), the result is as follows:

$$I(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta}) + D(\dot{\Theta}) + B_J\dot{\Theta} + K_j\Theta = B_J\dot{\Theta}_o + K_j\Theta_o + T_e \tag{19.10}$$

This controller implements in robot joint space a dynamic behavior analogous to that depicted in Figure 19.12; the controller spring and damper serve to push or pull the robot pose toward that specified by the virtual trajectory.

Due to the nonlinear kinematics of a typical robot, constant stiffness and damping matrices in the control law of Equation (19.9) result in stiffness and damping apparent at the robot's end-effector (the usual interaction port) that vary with the robot's pose. However, a straightforward extension of Equation (19.9) may be defined to control end-effector impedance, using the robot's kinematic transformations to express a desired end-effector impedance in robot joint space. The required transformations are the forward kinematic equations, denoted $X = L(\Theta)$, relating robot pose to end-effector position (and orientation), X; the pose-dependent Jacobian of this transformation, denoted $\dot{X} = J(\Theta)\dot{\Theta}$, relating robot velocity (e.g., joint rates) to end-effector velocity (and angular velocity); and the transformation relating end-effector forces (and torques), F_e, to actuator torques. For the common arrangement in which each robot actuator acts independently on a robot joint variable, the required transformation is the transpose of the Jacobian: $T_a = J^t(\Theta)F_e$. To achieve a target impedance with constant end-effector stiffness K and damping B, the control law is as follows:

$$T_a(x_o, \dot{x}_o, \Theta, \dot{\Theta}) = J^t(K(x_o - L(\Theta)) + B(\dot{x}_o - J(\Theta)\dot{\Theta})) \tag{19.11}$$

This is a nonlinear counterpart of the control law in Equation (19.9). An important point is that all of the required nonlinear transformations are guaranteed to be well-defined throughout the robot workspace — even where the Jacobian loses rank.[9] Thus the simple impedance controller can operate near or at the robot's kinematic singularities.

Note that the simple impedance controller does not rely on force feedback and does not require a force sensor. If the forces due to the robot's inertia and friction are sufficiently small, the robot's interaction port behavior will be close to the desired impedance. Inertial forces decline for slower movements and vanish if the robot is at a fixed pose. As a result a simple impedance controller can be quite effective in some applications. Frictional forces may also decline for slower movements but, especially due to dry friction, need not vanish when the robot stops moving. This is one reason why, for applications requiring low impedance, low inertia and friction are desirable in the design of interactive robots.

From a controller design viewpoint, the simple impedance control law of Equation (19.9) closely resembles a PD motion controller acting on the error between actual and virtual trajectory in joint space. Given the assumptions outlined above (a robot modeled as a mass driven by controllable forces) the impedance controller stiffness K and damping B correspond, respectively, to proportional and derivative gains. The nonlinear simple impedance controller of Equation (19.11) resembles a gain-scheduled PD motion controller in which the nonlinear transformations adjust the proportional and derivative gains to achieve constant end-effector behavior.

[9]The term *joint space* refers to a set of generalized coordinates of the robot mechanism, which uniquely define the configuration of the robot and hence the position and orientation of any interaction port. Similarly any force and torque applied to the interaction port can be projected to determine the corresponding forces and torques on the actuators.

The simple impedance controller has another important feature. Insofar as the controller exactly mimics the behavior of a spring and damper, the robot behaves exactly as it would with a real spring and damper connecting it to the virtual trajectory. If the virtual trajectory specifies a constant pose, the entire system (robot plus controller) is passive and, therefore, guarantees stable interaction with all passive environments. Regardless of the actual robot mass, damping, and controller gains, coupled instability is completely eliminated. In fact, this is true even if the interaction occurs at points other than the end-effector, whether inadvertently or deliberately (e.g., consider using an elbow or other body part to manipulate) (Hogan, 1988). In addition, because the simple impedance controller does not rely on force feedback control to shape impedance, it is not vulnerable to the loss of passivity that can occur when structural modes interact with a force feedback loop (as illustrated in Example 19.2).

Thus, although it is primitive, a simple impedance controller goes a long way toward solving the stability problem, and its performance gets better as the inherent robot impedance is reduced. In practice, though this implementation performs well in some applications, it has limitations. Several factors make the controller impedance nonpassive, including discrete-time controller implementation and unmodeled dynamics between actuator and sensor. Under these conditions stability cannot be guaranteed with all environments. At the same time, the creation of low-impedance hardware can be difficult, particularly for complex geometries. Still, the approach has been quite successful, particularly when used in conjunction with highly back-drivable designs (for example, using robots for physiotherapy (Volpe et al., 2000)). Feedback methods to reduce intrinsic robot impedance are discussed below.

19.4.3 Direct Impedance Modulation

A complementary approach to implementing interaction control without relying on feedback is to modulate impedance directly. For example, as discussed below, feedback control of interaction-port inertia can be especially challenging. However, apparent inertia is readily modulated by taking advantage of kinematic redundancies in the robot mechanism. Using the human musculo-skeletal system as an example, all aspects of mechanical impedance, including inertia, may be varied by using different body segments as the interaction port (e.g., the hand vs. the foot) and/or by varying the pose of the rest of the musculo-skeletal system. For example, with the phalanges partially flexed the fingertips present extremely low mechanical impedance to their environment, while with the elbow fully extended, the arm presents a high impedance along the line joining hand to shoulder.

An alternative way to modulate mechanical impedance is by taking advantage of actuator redundancies. Because muscles pull and don't push they are generally arranged in opposition about the joints so that both positive and negative torques may be generated. Consequently, the number of muscles substantially exceeds the number of skeletal degrees of freedom. Interestingly, the force developed by mammalian muscle varies with muscle length and its rate of change, giving it mechanical impedance similar to a damped spring, and this mechanical impedance increases with muscle activation. While the torque contributions of different muscles may counteract each other, impedance contributions of muscles always add to the joint impedance (Hogan, 1990). As a result, mechanical impedance about a joint may be directly modulated without adjusting pose by simultaneous contraction of opposing muscles. For example, a "firm grip" on a tool handle may be achieved by squeezing it. That requires simultaneous activation of opposing muscles (at least in the hand and forearm but likely throughout the upper arm) which can be done without altering the net force and/or torque on the tool, changing only the mechanical impedance of the hand where it interacts with the tool and, in turn, stabilizing the pose of the tool relative to the hand.

The human ability to modulate neuromuscular mechanical impedance at will by co-contracting muscles is readily observed, and human control of mechanical impedance appears to be quite sophisticated and selective. For example, recent experimental results (Burdet et al., 2001) have shown that human subjects making point-to-point movements holding the handle of a planar robot that presented a stabilizing impedance along the direction of motion and a destabilizing impedance in the normal direction increased their arm impedance preferentially along the normal but not the tangent.

A similar approach to direct modulation of actuator impedance may be implemented in robotic systems. There are many ways this may be accomplished. For example, a closed volume of air in a pneumatic actuator acts as a spring; because of the nonlinear relation between air pressure and volume or due to a change of shape, increasing pressure increases the apparent stiffness. Fluid flowing through a passage (e.g., in a hydraulic actuator) dissipates energy, and varying the geometry of the passage changes the apparent damping (Kleidon, 1983). Alternatively, field-responsive fluids (e.g., electro-rheologic or magneto-rheologic) permit variation of viscous damping without changing geometry and have been used in automotive applications (Carlson et al., 1999). Changing excitation current and hence magnetic field strength in an electromagnetic actuator (e.g., a rotary or linear solenoid) may be used to change both apparent stiffness and damping, and electromechanical actuator designs that provide for direct modulation of impedance have been proposed (Fasse et al., 1994).

To the best of our knowledge, actuators with directly-modulated impedance have yet to see widespread applications in robotics. However, the development of new actuator technologies to serve as "artificial muscles" is presently a topic of active research, and new applications may emerge from this work.

19.5 Improving Low-Impedance Performance

A large class of applications, including robots that interact with humans, demands interactive robots with low mechanical impedance. The most direct approach is to design low-impedance hardware and use a simple impedance control algorithm; in fact, this is the recommended approach. However, intrinsically low-impedance hardware can be difficult to create, particularly with complex geometries and large force or power outputs. Most robotic devices have intrinsically high friction and/or inertia, and the simple impedance control technique described above does nothing to compensate for intrinsic robot impedance. Considerable effort has been devoted to designing controllers to reduce the apparent endpoint impedance of interactive robots.

19.5.1 Force Feedback

Force feedback is probably the most appealing approach for reducing apparent impedance. If a simple impedance controller is applied to a one-dimensional inertial mass as in Figure 19.1, that is also subject to some nonlinear friction force $F_n(x, \dot{x})$, and is augmented with a proportional force feedback controller, the control law is given by Equation (19.6), repeated here for convenience.

$$F_a = K(r - x) + B(\dot{r} - \dot{x}) + K_f[F_e + K(r - x) + B(\dot{r} - \dot{x})] \tag{19.12}$$

K and B are the scalar desired stiffness and damping, respectively, and r now represents the virtual position.[10] The force feedback term serves to minimize the deviation of the actual endpoint force from the desired endpoint force, which looks like a damped spring characterized by K and B connected to the virtual trajectory. The equation of motion for the uncontrolled system with nonlinear friction is

$$m\ddot{x} + b\dot{x} + F_n(x, \dot{x}) = F_a + F_e \tag{19.13}$$

Combining Equation (19.12) and Equation (19.13), the controlled equation of motion is as follows:

$$\frac{m}{1 + K_f}\ddot{x} + \frac{b}{1 + K_f}\dot{x} + \frac{F_n(x, \dot{x})}{1 + K_f} + K(x - r) + B(\dot{x} - \dot{r}) = F_e \tag{19.14}$$

[10] As outlined above, given the assumed robot model, a simple impedance controller closely resembles a PD motion controller. In Equation (19.6) and Equation (19.14) the corresponding proportional and derivative gains are $K(1+K_f)$ and $B(1 + K_f)$, respectively. This form is chosen so that force feedback does not change the stiffness and damping introduced by the controller.

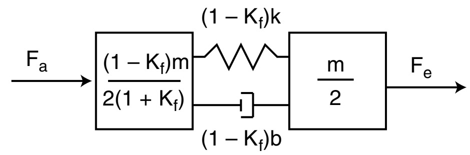


FIGURE 19.13 Equivalent physical system for a single-resonance model under force feedback control. (Modified from Colgate, J.E., The control of dynamically interacting systems, Ph.D. thesis, Massachusetts Institute of Technology, 1988, with permission.)

Equation (19.14) shows that the introduction of proportional force feedback reduces the apparent mass and friction by a factor of the force feedback gain plus one, so that (in principle) arbitrarily large force gains drive the actual impedance to the desired impedance specified by the controller. Friction, inertia, or any other behavior, whether linear or nonlinear, is minimized.

This one-degree-of-freedom combination of force and motion feedback is readily extended to multiple degrees of freedom. Assuming the robot may be modeled as a multi-degree-of-freedom inertial mechanism, retarded by friction and subject to actuator and environmental forces (as in Equation (19.8) above), a controller based on feedback of endpoint position, velocity, and force can be formulated to replace the manipulator's inherent dynamics with arbitrary inertia, stiffness, and damping (Hogan, 1985).

Despite its appeal, this approach has fundamental limitations. Clearly, the largest possible force feedback gain is desirable to minimize unwanted components of intrinsic robot behavior, e.g., due to nonlinear friction. However, experienced control system designers intuit that increasing gains toward infinity almost always leads to instability, and the following observation shows why this holds true in the case of force feedback. Colgate has derived a *physical equivalent representation* of force feedback. A physical equivalent representation is a model of a mechanical system that exactly replicates the behavior of a system under feedback control. Figure 19.13 shows a physical equivalent representation for a model of a robot with a single structural vibration mode (as shown in Figure 19.2 but with $b_1 = b_2 = 0$, $m_1 = m_2 = m/2$) under force feedback control (Equation (19.12) with $K = B = 0$). It also exhibits a single structural mode but with parameters that depend on the force feedback gain. The result has *negative* stiffness, mass, and damping parameters for any force feedback gain greater than 1 (Colgate, 1988). Because endpoint force feedback is not state feedback, *isolated* stability is unaffected; this system remains neutrally stable as long as the interaction force F_e is zero. However, for $K_f > 1$, it is not passive; it can exhibit negative visco-elastic behavior upon contact, resulting in coupled instability. Note that this gain limit is independent of the value of the structural stiffness and damping. As any real robot almost inevitably exhibits resonance, this analysis shows that any system under force feedback becomes nonpassive for a force feedback loop gain greater than unity and is, therefore, vulnerable to coupled instability.

19.5.2 Natural Admittance Control

One effective solution to the loss of passivity due to force feedback is natural admittance control (NAC) developed by Newman (1992). In essence, this approach is based on the observation that a system under force feedback becomes nonpassive when the controller reduces the apparent inertia (to less than about half its physical value), but passivity is not compromised by the elimination of friction. Natural admittance control specifies a desired inertia that is close to that of the physical system (the "natural admittance") and focuses on reducing friction as much as possible, preserving passivity.

A method for the design of natural admittance controllers is detailed in Newman (1992). Here the procedure is sketched for the simple robot model shown in Figure 19.1 consisting of a single mass retarded by friction and driven by actuator and environmental forces. The system might also have nonlinear friction that the NAC seeks to eliminate, that need not be modeled at all in the design of the compensator; the controller treats such friction as a disturbance from desired port behavior and rejects it. To ease notation, the substitutions $v = v_a = \dot{x} = v_e$ are made. The equation of motion for the system velocity neglecting nonlinear friction is

$$m\dot{v} + bv = F_a + F_e \tag{19.15}$$

In the Laplace domain:

$$v = \frac{1}{ms + b}(F_a + F_e) \tag{19.16}$$

A generic form for the controller is assumed, that incorporates some velocity feedback with compensator $G_v(s)$ and endpoint force feedback with some compensator $G_f(s)$:

$$F_a = G_v v + G_f F_e \tag{19.17}$$

Using Equation (19.16) and Equation (19.17) the actual endpoint admittance $Y(s) = v/F_e$ is determined:

$$Y(s) = \frac{G_f + 1}{ms + b - G_v} \tag{19.18}$$

This is equated to the target port admittance. The target stiffness K and damping B are chosen at will but the target mass is equal to the physical mass m of the system:

$$Y(s) = Y_{des}(s) = \frac{1}{ms + B + K/s} \tag{19.19}$$

Equation (19.18) and Equation (19.19) can be solved for G_f in terms of G_v:

$$G_f = \frac{(b - G_v - B)s - K}{ms^2 + Bs + K} \tag{19.20}$$

A simple form for G_v can be assumed, such as a constant, and the compensator design is complete. Equation (19.20) may be thought of as a force feedback "filter" that yields the desired admittance of Equation (19.19). Although the mass is not reduced, the "disturbance" to the desired behavior due to friction, b, is rejected with the feedback gain of the compensator, and its effect is smaller as the velocity loop gain, G_v, is increased. The approach serves equally well to minimize disturbances due to nonlinear frictional forces, e.g., dry friction.[11]

In principle, the controlled system is passive even if the velocity gain is increased to arbitrary size, minimizing the unwanted frictional effects. In practice unmodeled dynamic effects limit the gain that may be used without compromising passivity, but the technique affords significant practical improvement over simple proportional force feedback. A more general formulation and discussion of the method can be found in Newman (1992) and Newman and Zhang (1994).

19.5.3 Series Dynamics

An alternative approach to stabilizing interaction under force feedback is to place compliant and/or viscous elements in series between a manipulator and its environment. Variants of this approach have been used since the earliest attempts at robot force control (Whitney, 1977), e.g., in the form of compliant pads on the robot end-effector. A drawback is that this may compromise fine motion control. More recently a series spring has been incorporated within force control actuators to facilitate force feedback and absorb impacts (Pratt and Williamson, 1995). The following example illustrates the beneficial effect of series dynamic elements.

[11]The Laplace domain representation above was used to simplify the presentation and is not essential.

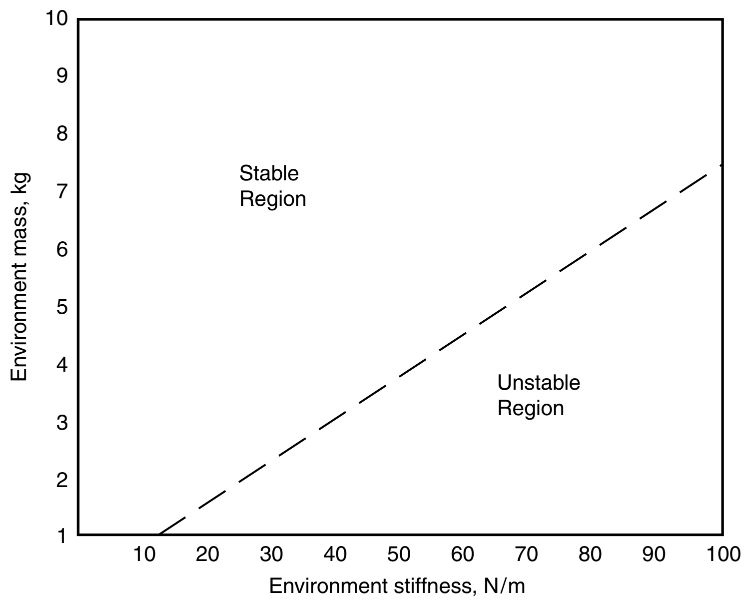


FIGURE 19.14 Stable and unstable parameter values for spring-and-mass environment coupled to the system of Example 19.2. The coupled system is stable when parameters fall in the region above the line and unstable when they fall in the region below the line.

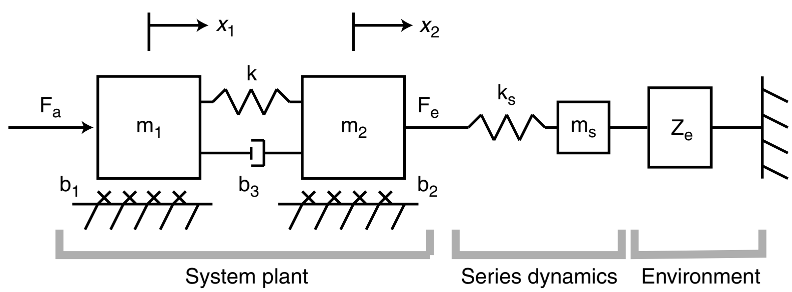


FIGURE 19.15 The single-resonance model of Figure 19.2 with a spring and mass interposed in series between interaction port and environment.

Example 19.5

A series spring's stabilizing effect on a force-controlled system can be illustrated by considering again the simple model of a robot with a single structural vibration mode shown in Figure 19.2. Under the control law of Equation (19.6) or Equation (19.12), this system may have a nonpassive impedance and be destabilized by interaction with some environments. However, it remains stable when interacting with a broad range of spring and mass environments,[12] as depicted by the stability boundary in Figure 19.14. In this particular example, high-stiffness springs are destabilizing while masses are stabilizing. If a spring and mass are placed in series between the interaction port and the environment, as depicted by k_s and m_s in Figure 19.15, the apparent stiffness that the controlled system experiences cannot exceed k_s. By appropriate

[12]Recall that the set of all positive springs and masses includes the most destabilizing passive environments a robot may encounter.

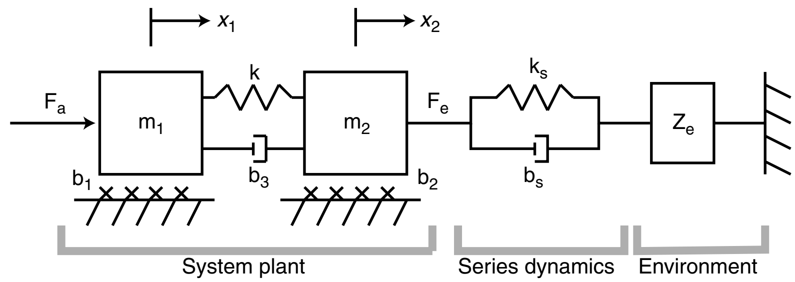


FIGURE 19.16 The single-resonance model of Figure 19.2 with a parallel spring and damper interposed in series between interaction port and environment.

choice of k_s the resulting interaction port impedance is made passive, stabilizing the interaction. In effect, a piece of the controller's "environment" is carried around with the robot, acting to "filter" the environment dynamics to ensure that the controller cannot encounter any environment capable of destabilizing it.

The implementation of "series elastic actuators" (Pratt and Williamson, 1995) also includes an inertia m_s between k_s and the environment, endowing the controller's environment with not only a maximum stiffness but also a minimum inertia, further stabilizing the system. Considering Figure 19.15, the stiffness k_s and inertia m_s may be chosen so that the uncoupled robot behavior corresponds to a point in the stable region. Assuming interaction with passive environments, the interaction port is then guaranteed to encounter only environments that reside above and to the left of this point, thereby ensuring coupled stability.

Although the method described above can guarantee stability for certain types of nonpassive systems, it lacks the generality to be applicable to all nonpassive systems. Dohring and Newman have provided a formalization that includes damping to drain the energy introduced by nonpassive behavior of the controller or the environment (Dohring and Newman, 2002). The recommended approach includes a parallel spring and damper in series with the environment, as shown in Figure 19.16. Analysis of passivity of the admittance function at the interaction port dictates the selection of the spring and damper values. By modeling the parallel spring and damper as a two-port, the admittance at the outboard side of the spring and damper, $Y(s)$, is computed in terms of the original robot admittance, $Y_o(s)$, and the series elements:

$$Y(s) = Y_o(s) + \frac{s}{b_s s + k_s} \tag{19.21}$$

A small series damper b_s contributes a large positive-real term to the net admittance at high frequencies, compensating for negative-real, nonpassive behavior in $Y_o(s)$. The spring determines the frequency range at which the damping dominates. Together they can be selected to make $Y(s)$ passive.

Intentional series dynamics can be thought of as "mechanical filters," perhaps loosely analogous to anti-aliasing filters used with analog-to-digital converters, that attenuate undesirable effects in peripheral frequency ranges that make the system vulnerable to coupled instability, while preserving desired dynamic behavior in the frequency range of interest. As with signal filters, something is lost; here, it might be bandwidth or positioning accuracy, a tradeoff that may be acceptable for interactive robots.

19.6 More Advanced Methods

The foregoing considered some of the basic approaches to interaction control. Alternative and more advanced techniques and applications have been developed. The following is a small sample of this extensive work.

Nonlinear approaches to impedance control have been pursued with success, including adaptive (Singh and Popa, 1995), learning (Cheah and Wang, 1998), robust sliding-mode (Ha et al., 2000), and decentralized